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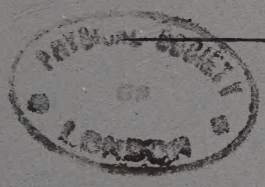

VOL. V, No. 5

JUNE 1922

KYOTO

Observations of Meteors Probably Connected With the Pons-Winnecke's Comet in 1921.

By

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(Received December 17, 1921)

§1. Introduction.

The return of the Pons-Winnecke's comet in the present year aroused the special attention of meteor observers expecting to catch fine displays connected with the comet during the latter part of June. The matter was first introduced by Mr. Denning to the world, early in this year, to the effect that not only the meteors but the comet itself was thought to be of exceptional interest because of the great popularity, probably due to some misunderstanding of the matter, since sometime before its re-discovery by Prof. Barnard on April 10, 1921. Mr. Denning had seen meteors of this kind in 1916, whose own notes* we quote here:

"The meteors of Pons-Winnecke's Comet are due to appear on or about June 26-28, and there is the prospect of a striking display this year, as the parent comet will be near perihelion. There was a shower (probably the first observed from this comet) in 1916 June 28, when it was witnessed from Bristol, after 10h 25m, under rather unfavourable conditions. Half an hour later it was detected at Bournemouth and Birmingham. The meteors of this stream are individually not very noteworthy, but they are fairly bright, slow object, with short paths in the early part of the night, the radiant being not far W. of the zenith.

* Observatory, 44, 61. (1921).

"In 1916 the radiation seemed very diffused over the region of Ursae Majoris, Quadrans and Bootes. This shower should be very carefully observed in all its details next June. A special degree of interest attaches to the system, as it is quite a modern development, and owes its occurrence to the accumulated perturbations by Jupiter on one of the short-period comets. The orbit of this particular comet, though now nearly intersecting the Earth's path, was formerly placed at a distance sufficiently great to enable its particles (or debris) to avert collision with, and destruction by, our atmosphere."

According to Mr. R. T. Crawford's elements of the comet, which are based upon the observations of April 12, 19, and 29 of the present year:

$$
\begin{aligned}
T &= 1921 \text{ April } 12.95, \text{ G.M.T.},\\
\omega &= 170^\circ\ 34'\\
\Omega &= 97\ \ 51\\
i &= 18\ \ 50\\
e &= 0.6779\\
\text{Period} &= (5.80), \text{ assumed},\\
q &= 1.040,
\end{aligned}
$$

the earth was to pass the descending node of the cometary orbit on June 29 on or later, which was a little more than two weeks after its perihelion passage.

For the observation of the meteors we were making some preparations since the early part of June, preparing the necessary star maps and watching the weather conditions. It was to our deep regret that the time for our venture was the rainy season of our country, which comes on annually, beginning in the early part of every June, and lasting about one month in almost the whole area of central Japan. The northern part of the country, especially Hokkaido, is almost wholly excluded from the influence of the rain, so that that region was thought to be the best place for our astronomical observations of the time, whither, however, some observers from the Tokyo observatory were reported to be sent; accordingly we decided to remain here, in the "western" part, for our work in spite of some seeming disadvantages in weather.

§2. Method of Observations.

For our observations, the star maps of gnomonic projection were used. Each sheet covers a diametral arc of about 50° in all directions, and consequently the whole area of the celestial sphere is covered by 20 sheets in all, including abundant duplications with neighbouring ones in their peripheries, the depths of the duplication being about 20°.

During the observations, the time of the Central Standard Time of Japan, referring to the 135th meridian E. of Greenwich, was kept, and that with pocket watches, the rates of which being adjusted daily by the clocks of our observatory. In distant fields, as described in the following pages, the time of the post offices or that of the railway station near by was kept, since these are to be trusted within a minute usually in our country.

As for the observers, who are also the present writers, one (I. Y.) of them is near-sighted, compensating his natural power with a pair of spectacles to the usual ability. The another (K. N.) has good eyes, and rather an exceptionally sensitive efficiency for faint lights, which is sufficiently proved by the fact that he can see twenty stars in the Pleiades on a clear night. This rare circumstance has effected the present richness of our harvest.

§3. Plan and Programme of Observations.

As mentioned above, the latter part of June is still within the rainy season in Japan. Although the mean state of rainy weather was not experienced *in the present year* owing probably to some extraordinary meteorological conditions, it being relatively dry since the beginning of the month, yet constant attention toward the sky was necessary. A special watch upon the barometrical distributions of the atmosphere was kept with direct observations as well as by the weather charts reported daily by the central and the local authorities of meteorological observatories, whether visits were frequently paid.

After the middle of June, clouds prevailed over western Japan caused by a train of cyclones coming from central China. On the morning of the 25th, however, the writers caught a gleam of hope of clear weather which might come for a day or two at least follow-

ing a receding cyclone. Then we decided to follow closely on the heals of the cyclone, and in the afternoon we started from Kyoto Railway Station eastward.

One of us (K. N.) got an observing station by the kindness of Mr. E. Yoshida, in Hachiman, Omi Province, about ten miles NE of Kyoto. Here the weather was pretty good before sunset, but afterward SK-clouds appeared from the western sky, and night observations became almost hopeless, although the keen eyes of our observer indeed caught several glimpses of the meteors.

In the same afternoon, the other observer (I. Y.) extended his journey beyond Hachiman, hoping, if possible, to get a station at Tsuruga. But noticing on his way dirty weather coming on, he got off his train at Hikone, Omi Province, where he visited the meteorological observatory there immediately, and found that a new cyclone was speedily approaching, which in fact betrayed us all. In Kyoto Mr. R. Furukawa and others were watching the sky the same evening to catch the meteors if possible, and they had their useless vigil for their pains!

The next night, the 26th, the sky was less cloudy in many places near Kyoto, only covered with thin veils of cloud and several observers watched the meteors. But they gathered no crop. Mr. Nakamura stayed the same night still in Hachiman, and through gaps in the clouds, he happily got many stars, among which he actually found a number of the expected Winnecke Meteors for the first time.

The report of this discovery of the long-waited for meteors by Mr. Nakamura awakened a new impulse among our co-labourers on the morning of the 27th, and the next plan was made. After examining the annual results of meteorological observations many years ago, we learned that the San-in region, that is, along the coast of the Sea of Japan in the western part of the Main Land of Japan, was comparatively poor in mean quantity of precipitation. Therefore our most hopeful field was taken there. In the afternoon of the 27th, both writers started for the San-in; on the way thither, we fixed on a station at Fukuchiyama, a town about twenty miles NW of Kyoto, for our observations of the night. Mr. Y. Iwamori was so kind as to receive us and invited us to make use of the grounds of the Middle School, which was under his management, and proffered many facilities for which we were very thankful. At this place the weather was not perfectly clear, but during the moments of clearness we got

many meteors and fully confirmed the apparitions of the Winneckes.

Soon after midnight, we said good bye to Fukuchiyama, to continue our westward journey. On the morning of the 28th, we arrived at Tottori, and there we visited the Prefectural Normal School in the town, whose Principal, Mr. T. Idé, kindly offered the school ground, for our observation post. Here we stayed four days, two of which were entirely rainy, accompanied by storms. The other two days were also constantly threatened by clouds, whose temporary gaps, however, were diligently caught.

On the morning of July 2nd, we started from Tottori on our way back to Kyoto, because the climax phase of the meteoric display was ended. Our original plan was to get a night's observation at some place near Himeji, ten miles W. of Kobe. But rain prevented us.

After our arrival at Kyoto, a constant watch of careful observations of the meteors was kept mainly by Mr. Nakamura on the observatory grounds every night of sufficient clearness.

After the middle of July, we two made an excursion to a distance, which interrupted our observations of the heavens. After returning home, however, Nakamura found that the Winnecke meteors were still visible, radiating from displacing radiant points.

August 8th was our last night.

§ Results of our Observations.

In the following table, the results of the meteoric observations are arranged. Here, not only those of the Winnecke's Comet, but all others observed at the same time are included.

Let us now note some abbreviations used. The *weights* of individual observations are described by an arbitrary scale from zero to 10, where zero is the extremely poor, and 10 the best. As to the *velocity*,

vR : very rapid ;
R : rapid ;
ʳR : rather rapid ;
M : moderate velocity ;
ˢS : rather slow ;
S : slow ;
vS : very slow.

For the *colours*,

B : Bluish,
BW : Bluish-white,
W : white,
Y : yellowish,
YB : yellowish-blue,
RY : reddish-yellow,
C : Crimson.

For the *observers*, those marked with an asterisk* on the No. are stars of I. Yamomoto, and others of. K. Nakamura.

In the last column, R, those of Remarks,

W : meteor probably belonging to Winnecke's Comet,
A : meteor of α Bootids,
B : meteor of η Draconids,
C : meteor of Ophiuchids,
D : meteor of ε Bootids.

The approximate terrestrial coordinates of the observing stations are :

Stations.	Longitude E. of Greenwich	Latitude.
Kyoto University Observatory	$9^h\ 3^m\ 7^s$	$+35^0\ 1'.6$
Hachiman, Omi	9 4 20	+35 8
Fukuchiyama	9 0 36	+35 18
Tottori	8 56 56	+35 30

TABLE I.

Results of Observations of Meteors.

June 25, at Hachiman.

No.	Time.	Appearance.	Disappearance.	Weight.	Duration.	Magn.	Velocity.	Color.	R.
	h. m.	o o	o o		s.				
1	8 32.8	207.5+13	200.5+24.5	4	0.2	5	R		
2	39.8	212.5+19.5	205 +14.5	5	0.3	5	rR		A
3	40.2	202 +12.5	206 +24	3	0.2	5	R		
4	45.0	212 +19.5	205.5+21	4	0.2	4.5	rR		A
5	9 11.4	212 +20	207 +21.5	5	0.2	5.5	rR		A

June 26, at Hachiman.

1	8 34.2	213.5+46	225 +47	7	0.2	6	R		
2	35.4	211 +55	175.5+48.5	6	0.2	5.5	R		
3	37.1	165 +46	159 +55.5	8	0.3	5.5	R	B.W.	
4	39.0	201.5+48.5	199.5+58.5	6	0.2	6	R		
5	39.8	209 +50	199.5+58.5	5	0.2	5.5	R		W
6	40.7	211 +51.5	209 +52	7	0.2	5.5	R		W
7	42.2	211.5+51.5	211 +60	6	0.2	6	rR		W
8	43.4				0.3	5	rS	B.G.	
9	44.2	204 +50	242 + 7.5	5	0.2	6	M	B.W.	
10	47.0	239 +0	242 + 7.5	6	0.3	5	M	B.W.	W
11	47.8	209.5+50	216.5+45.5	5	0.2	4.5	M	W.	W
12	49.6	212 +53	207.5+56.5	8	0.2	5.5	M	B	W
13	49.6	210 +49	211.5+50	8	0.3	5	M	B	W
14	52.0				0.2	5.5	M		
15	52.8	213 +45.5	215.5+37.5	8	0.2	5.5	M		W
16	54.4	189 +54	181.5+59	6	0.3	5.5	M		A
17	55.8	219 +41.5	205 +11.5	9	0.5	5	R	B.W.	B
18	58.0	204.5+43.5	204 +32	6	0.2	5.5	rR		
19	58.8	206 +62	185 +60	8	0.3	4	R	B.W.	B
20	9 0.9	232 +35	227.5+41	7	0.2	6	rR		W
21	2.8	239 +22.5	235.5+30.5	7	0.2	5.5	rR		
22	4.9	235 + 6.5	137.5+ 3	7	0.2	6	rR		
23	12.0				0.3	5	rR		
24	14.2	201 +53	198 +53	5	0.3	5.5	M		A
25	15.8	198.5+41	202.5+22.5	7	0.3	5	vR		
26	16.8	209 +51	205.5+54.5	6	0.2	5.5	rS		W
27	18.2	236 + 8	241.5+65	6	0.2	5.5	R	B.W.	
28	21.2	212.5+49.5	214 +43.3	6	0.3	5.5	rR	B.	W
29	22.8	218.5+43.5	222.5+38	5	0.2	5.5	rR	W.	W
30	25.0	217 +33	225 +21	8	0.4	5	rR	W.	W
31	31.8	214.5+43.5	217.5+35.5	6	0.3	5.5	rR		W
32	10 1.2				0.3	4.5	R	B.W.	

June 27, at Fukuchiyama.

No.	Time.	Appearance.	Disappearance.	Weight.	Duration.	Magn.	Velocity.	Color.	R.
	h. m.	o o	o o		s.				
1	9 6	208 +50.5	204.5+55	6	0.2	5	↗R		W
2	8.2	211 +33.5	211 +17	5	0.2	5	↗R	B.W	W
3	11.1	225 +27.5	223 +32	8	0.2	5	R		
4	12.3	211.5+51.5	222.5+45	8	0.3	4.5	R	B	W
5	14.7	196 +62.5	214 +65	8	0.3	5.5	R		
6	15.0	220 +49	210.5+55.5	7	0.2	5	R	B	
7	15.6	210.5+51.5	219 +54.5	5	0.2	5.5	↗R		W
8	18.1	262 +6	262 − 2	5	0.5	1.5	↗R		
9	19.7	227 +42.5	217 +46	3	0.3	5	M		
10	20.8	212.5+48.5	227.5+40	9	0.4	4.5	M	B.W	W
11	23.0	205 +38	201 +33.5	7	0.2	5.5	R		W
*12	23	200 +45	187 +36	8	0.1	4	R	W	
13	24.8	208.5+50	208.5+62	3	0.2	5.5	R		W
14	26.3	204 +46	199.5+54	8	0.2	6	VR		A
15	28.0	224.5+41	203.5+33	4	0.2	5	R		B
16	30.2	217.5+41	220.5+35	6	0.3	5.5	M		W
17	31.2	211.5+47.5	208 +38.5	2	0.2	5.5	↗R		W
18	33.7	201 +43	192.5+36	8	0.2	5.5	R		W
19	33.8	216 +54	229 +58	7	0.3	5.5	↗R		W
20	35.2	231 +61.5	192.5+51	7	0.2	4.5	VVR	W	
21	37.0	209 +52	214 +57	7	0.2	5	R		W
22	37.8	212 +50	220.5+49.5	7	0.2	5	R		W
23	39.8	227.5+42.5	211 +47	8	0.3	5	R		
24	40.8	210 +70.5	228.5+76	9	0.2	5	R		
25	42.8	211 +51	222 +38	6	0.2	5	↗R		W
26	44.0	262.5+79	7 +82.5	8	0.3	4.5	VR		B
27	45.0	221 +52	248 +59	6	0.3	5	R		
28	45.8	212 +53	230 +60	4	0.2	5.5	R		W
29	47.1	222 +41	227 +34.5	6	0.3	5	R		W
30	49.7	209.5+51	207.5+57	8	0.2	5	↗R		W
31	51.8	204.5+56	194.5+63	3	0.2	5	↗R		W
32	10 36.7	216 +70	222.5+76.5	5	0.3	3.5	↗R		W
33	49.5	213 +24	216 +34	8	0.3	3.5	R	G.W.	A
*34	11 1	277 +75	272.5+35	6	0.2	3	R	W	
35	1.5	283.5+46	277 +33.5	3	0.2	4	R		
*36	8	287 +45	285 +34	7	0.1	3			
37	11.8	224 +76	248 +71	5	0.2	4	R		
38	14.0	211 +51	216.5+39.5	8	0.4	4	R	B	W
39	16.8	231 +59	229.5+51	5	0.3	5	↗R		
40	18.7	129 +81.5	220 +75	6	0.3	5	R		
41	20.2	208.5+52	212 +57	5	0.2	4.5	VR		W
42	22.0	255 +71.5	285 +73	5	0.3	4.5	R		W
43	23.5	212 +51.5	225 +52.5	6	0.2	5			W
44	26.5	267.5+32.5	279 +33	6	0.2	4	R		A

June 28, at Tottori.

No.	Time.	Appearance.	Dis-appearance.	Weight.	Dura-tion.	Magn.	Velo-city.	Color.	R.
	h. m.	o o	o o		s.				
1	8 47.7	205 +49	212.5+57	8	0.3	5.5	rR	W	W
2	48.2	215 +42.5	220 +35	7	0.3	5	R	B.W.	W
3	49.2	207 +58.5	198 +67.5	7	0.4	5.5	rR	W	W
4	50.0	210.5+51.5	206 +54.5	9	0.2	5.5	rR		W
5	50.8	212.5+48	224 +43.5	7	0.4	5	rR		W
6	53.0	169 +16	162.5+26.5	9	0.6	5	R		
7	53.7	192.5+39	185 +42	10	0.3	5.5	rS	W	A
8	54.8	212.5+46.5	217.5+36	10	0.4	5.5	rR	B.W.	W
9	56.0	201 +32	195.5+22.5	7	0.5	5.5	rR	B.W.	W
10	57.2	210.5+65	204.5+56.5	10	0.4	5	M	B.W.	
11	57.5	194 +41	192.5+31	4	0.4	5.5	S		
12	58.2	208 +25	207.5+17.5	8	0.3	5	M	W	W
13	59.0	158 +61	168 +54.5	7	0.4	5.5	—	B.W.	
*14	59.0	219 +77.5	214 +72.5	7	0.2	2	Y	W	
15	59.8	211 +51	206 +58	6	0.4	4.5	rS	B	W
16	9 0.7	213.5+51.5	219 +55	6	0.2	5	M	W	W
*17	1	122.5− 5	127.5− 9.5	5	0.5	2	W	B	
18	1.5	209 +51	206.5+55	5	0.2	5	M	B.W.	W
19	2.4	242 +56.5	275 +72.5	10	0.6	4	vR	W	
*20	3	252 +58	236 +65	8	0.3	4	W		
21	3.7	218 +45	227.5+37.5	8	0.4	5.5	rR	W	W
22	5.0	211.5+48.5	216 +42.5	5	0.2	5	rR	W	W
23	5.7	212.5+48	217.5+44	7	0.3	4.5	rR	B.W.	W
24	6.8	213 +50	216.5+57	7	0.3	5.5	rR	B.W.	W
25	7.8	212.5+49	209 +56	9	0.3	5.5	rR	W	W
26	8.5	209 +42	206 +32	8	0.5	5	rR	B.W.	W
27	9.1	207.5+29.5	206 +20	7	0.5	5	rR		W
28	10.4	251 +68	225 +71.5	9	0.4	4	M	B.W.	
29	11.2	214.5+54.5	219.5+61.5	7	0.4	5	M	B.W.	W
30	11.8	212.5+48.5	214 +40	7	0.4	5.5	M	B.W.	W
31	12.9	206.5+52	200 +57	8	0.3	5	rR	W	W
*32	13	177 +59	171 +59	6	0.2	5			
33	13.8	208 +48	203 +40	7	0.4	5	rR	W	W
34	14.7	204 +54	195.5+57.5	8	0.3	5	rR	W	W
35	15.3	211 +52	212 +58	6	0.3	5.5	rR	B.W.	W
36	16.3	168 +53	155 +59	7	0.5	5.5	rR		
37	38.3	209 +51.5	207 +56	8	0.3	6	R		W
*38	39	167 +54	151 +51	8	0.3	2	Y	W	
39	41.2	173 +43	154 +40	4	0.6	3	M	W	W
40	46.8	213.5+29	214 +17	8	0.5	2	rR		W

July 1, at Tottori.

No.	Time.	Appearance.	Disappearance.	Weight.	Duration.	Magn.	Velocity.	Color.	R.
	h. m.	o o	o o		s.				
0	8 58	213 +46.5	217 +42	5	0.2	5.5	R		W
1	9 2	229 +29.5	232.5+24	5	0.3	4.5	rR	R	W
2	2	230 +60	227.5+71	3	0.4	4	R		
3	31	209.5+52	203.5+66.5	4	0.6	3.5	R		W
4	35	237.5+32.5	227 +21.5	5	0.5	3	rR		
5	37	234 +29	221 +31	5	0.6	4	R		
6	10 28	211 +24	212 +17.5	2	0.4	2	R		W
7	32	206 +52.5	193 +56.5	3	0.4	2.5	R		W
8	38	201 +42.5	192.5+34.5	3	0.5	3	R		W
9	40	181 +56.5	166 +59	4	0.4	3	R		W
10	41	192 +55.5	183 +58	4	0.2	4	rR		W
11	42	212.5+48.5	216.5+58.5	5	0.4	3	R		W
12	42.5	212.5+47.5	217.5+43	5	0.3	3.5	R		W
13	43	212.5+48.5	210.5+55.5	5	0.4	4	R		W
14	43.7	201.5+56.5	193 +62	4	0.2	4	rR		
15	44.8	212.5+47.5	217 +40.5	5	0.4	3	R		W
16	46.2	217.5+28.5	213 +20.5	6	0.5	3	rR		B
17	48.4	211 +47	210.5+42.5	2	0.3	3	M		W
18	50.3	207 +52	201.5+55	2	0.3	3.5	M		W
19	51.0	216 +84	28 +87	3	—	4	—		W

July 3, at Kyoto.

1	8 31.6	170° +56	165 +62	6	0.3	5.5	rR	Violet	
2	32.2	206.5+50	211 +42	7	0.4	4.5	rR	R	
3	32.8	172 +50	163 +48	8	0.3	5.5	R		W
4	33.5	194 +56.5	197 +61.5	5	0.4	5	M	RW	
5	34.2	200 +55.5	208.5+59	8	0.3	5.5	M	BW	
6	34.8	161 +59.5	166.5+54.5	6	0.3	5.5	M		
7	35.3	184 +66.5	169 +70.5	6	0.3	5.5	rR	W	W
8	37	196 +55.5	190 +64.5	5	0.4	5.5	M		
9	37.6	213.5+46.5	215.5+43	7	0.2	5.5	R		W
10	39.2	176 +54.5	163 +54	5	0.4	4	M	W	W
11	40.2	169.5+61	163.5+57	5	0.2	5	R	R	
12	41	171.5+41.5	162.5+47.5	7	0.3	5.5	rR	W	
13	42	176 +55.5	175 +61	4	0.3	5	R	RW	
14	42.2	164 +57	172 +58.5	7	0.2	5 5	rS		
15	42.7	175 +48.5	169.5+52.5	6	0.3	5.5	R		
16	44	199.5+52	187.5+54	3	0.3	3	R		W
17	45	168 +56	165 +62	3	0.2	5.5	rR		
18	46	199.5+56	194 +59	7	0.2	6	R		W
19	46.8	206 +50.5	201 +57.5	7	0.3	5	R	W	
20	48.2	202.5+52.5	195 +56.5	8	0.3	5	rR		W
21	49	191.5+57	180 +62	7	0.2	6	R		W

No.	Time.	Appearance.	Dis-appearance.	Weight.	Dura-tion.	Magn.	Velo-city.	Colour.	R.
	h. m.	° °	° °		s.				
22	8 51	202 +54	190 +55.5	6	0.3	5.5	*r*R		
23	53	233 +58.5	244 +61.5	8	0.2	5.5	R		W
24	56	204 +50	194 +40.5	6	0.4	5.5	R		
25	58	192.5+56.5	194 +51.5	6	0.4	6	*r*S	W	
26	58.9	209 +51.5	204 +57	8	0.3	5	*r*R	BW	W
27	59.8	211.5+51	220 +64	7	0.4	3	*r*R		W
28	9 1	269 +44	279 +40.5	5	0.3	4	*r*R		W
29	1.9	221 +64.5	230 +71	7	0.4	5	*r*R	W	W
30	3.1	214 +46.5	217.5+42.5	6	0.2	5.5	*r*R		W
31	3.8	207.5+51	209 +56.5	5	0.2	5.5	M	RW	
32	5.7	171 +26	176 +17	4	0.2	5	R		
33	10.6	214 +48	220 +48	7	0.3	5.5	*r*S	W	W
34	12.2	272 +86.5	0 +89	8	0.2	5	S	RW	
35	13.2	209 +47.5	202.5+48	7	0.3	5	S	W	W
36	14.8	208 +53.5	200 +64.5	6	0.3	5	R	W	W
37	15.2	207.5+51	202 +56	7	0.3	5	M	RW	W
38	22	211 +52	212.5+59.5	7	0.4	5.5	M	W	W
39	22.8	256.5+18	262.5+13	5	0.4	4	M	Y	W
40	24	211 +48	201 +51.5	7	0.3	5	*r*R	Y	W
41	36	228.5+48	207 +47.5	8	0.4	3	R	Y	
*42	36	205 +47	192.5+42.5	8	0.2	4	R	Y	W
43	42	236 +37.5	231 +22	8	0.6	4	*r*S	WY	B
44	46.2	212.5+45.5	213.5+15.5	9	1.0	5	R	W Streak	W
47	50.8	212.5+49	206 +60	8	0.4	5	*r*R	W Streak	W
48	52.3	230 +58	242.5+37.5	6	1.0	2	*r*S	Y	
49	54.8	213.5+47.5	214 +44.5	8	0.2	2.5	*r*S	Y	W
50	56.2	219 +47	212.5+52	7	0.3	4.5	M	BW	
51	10 2	181 +25	177.5+16	6	0.4	4.5	M	BW	W

July 4, at Kyoto.

1	8 38.2	168 +62.5	165 +57.5	7	0.2	5	R	Y	
2	39	165 +59	155 +56	4	0.2	4.5	R	B	
3	40	207.5+50.5	202 +55	8	0.3	5	M	W	W
4	40.2	152.5+55.5	146 +60	7	0.2	5.5	M	B	
5	41.4	152.5+68.5	164 +59	8	0.4	4	M	RY	
6	42.7	163.5+62	175 +67	6	0.2	5.5	*r*R	W	
7	44.3	212 +47.5	216 +41	8	0.3	4	M	B	W
8	48.4	217 +38.5	219.5+34	7	0.2	5	*r*R	W	W
9	50.4	205 +50	196.5+52.5	7	0.2	5	R		W
10	52.5	219.5+13.5	207.5+18.5	7	0.5	2	R	RY	C
11	58.4	227 +33	223 +40.5	8	0.3	5	S		
12	9 0	212 +47.5	200 +51.5	9	0.4	4.5	R		W
13	8	220.5+27	230.5+16	7	0.5	5	R		
14	11.2	232.5+10.5	225 + 7	7	0.2	5.5	R	Y	

No.	Time.	Appearance.	Disappearance.	Weight.	Duration.	Magn.	Velocity.	Color.	R.
	h. m.	o o	o o		s.				
15	12.8	245.5+ 3.5	236 +14.5	9	0.4	5	↗R	B	C
16	13.6	238 −26.5	229.5−30	8	0.2	5	M	W	
17	14.8	239 −19.5	247.5−10.5	9	0.4	5	M	W	
18	15.7	240 +23	232.5+18	7	0.2	4	↗R		
19	16.2	247.5+22	239 +26.5	8	0.3	4	M	WB	
20	17.4	265.5+ 2	260 − 4	6	0.3	4	M	R	
21	18.3	237 + 6.5	245 +11	7	0.3	5.5	R	W	
22	20.2	256 + 9	260.5+12.5	7	0.3	4.5	R		
23	21.4	244 +19.5	246.5+13	7	0.2	4.5	↗R	W	
24	23.0	235 + 8.5	232.5+12.5	6	0.2	5.5	R	B	C
25	24.7	263 +12.5	270 +10	7	0.2	5	↗R	WY	
26	26.8	245.5− 8	240.5−12	7	0.2	5	↗S		C
27	28.5	234 + 7.5	232.5+10.5	7	0.2	5.5	R	W	
28	30.0	235.5+ 2.5	246 − 2.5	6	0.2	5	↗R	W	
29	31.8	242.5− 3.5	246 − 2.5	6	0.2	5	↗R	W	
30	33.2	240 −26	249 −26	6	0.2	4	R		
31	36.8	235.5+15.5	241 +16	5	—	5	↗R	Streak	
32	38.0	224.5+27	217 +24.5	5	0.3	5	S	W	
33	41.0	235.5+ 9.5	239.5+ 4.5	7	0.2	5	R		
34	43.0	234.5+11	245 5+12	8	0.2	5.5	↗R		
35	45.1	252.5+ 9	246 + 6	7	0.2	5	S	W	
36	49	253.5− 5.5	249 −14.5	8	0.5	4.5	S	Brown	
37	10 22.2	213.5+ 1.5	215 +55	7	0.3	4.5	R	B	W
38	24	213 +48.5	213.5+53	8	0.2	5	R		W
39	28.5	222 +32	218.5+40	4	0.2	4.5	R	W	
40	33.0	219 +12.5	207.5+17.5	8	0.5	4	↗R	Y	C
41	35.3	213 +46.5	226.5+47.5	7	0.2	4.5	R	W	W
42	36.5	207 +26	205 +18.5	6	0.4	5	R	BW	W
43	38.0	226 +20.5	218.5+27	6	0.4	5	R	YW	C
44	39.7	225 +42	235 +38	8	0.2	4.5	R	BW S.	W
45	41.7	214 +48	218 +52.5	8	0.3	5.5	R	B	W
46	43.3	213.5+36.5	213.5+31.5	7	0.2	5	M	W	W
47	44.1	212 +48.5	204 +58	7	0.4	4.5	↗R	B	W
48	51	211.5+56	202 +47.5	7	0.4	3	R	RY	

July 5, at Kyoto.

1	8 48	226 +40	212.5+39	7	0.6	5	S	B	
2	48.6	213 +48	212 +54	8	0.3	5	M	W	W
3	49.6	180 +52	173 +58	7	0.2	5.5	↗R	W	
4	48.6	195 +39	200 +42	8	0.2	5.5	R	BW	
5	52	250 +21	252.5+10.5	5	0.2	4.5	↗S	R	
6	53	214 +48.5	216 +53	8	0.3	5.5	↗S	YB	W
7	54.8	228.5+33	224 +41	7	0.5	5	↗S	W	
8	57	230.5+60.5	229 +69	8	0.6	5.5	M	—	

No.	Time.	Appearance.	Disappearance.	Weight.	Duration.	Magn.	Velocity.	Color.	R.
	h. m.	o o	o o		s.				
9	8 58	205 +48.5	194 +51	6	0.2	5.5	S	—	W
10	59	205 +49.5	192 +52.5	6	0.2	4	S	Y	W
11	9 1	213 +38	213.5+34	6	0.2	5.5	M	B	W
12	2.1	214 +43	217 +47.5	7	0.3	3	↗R	B streak	W
13	3.3	213 +47	214 +55	—	0.4	5	—		W
14	4	213 +35.5	204.5+32	6	0.4	2	R	R.Y.	
15	10	207 +27	172 +37.5	6	0.5	4.5	R	Y	C
16	17	194.5+68	176 +76	7	0.4	4.5	↗R	B	W
17	20	224.5+41	233 +36.5	6	0.3	5	↗R	B.W.	W
18	21.6	200 +55.5	193 +59	6	0.3	5	↗R	B streak	W
19	23.2	194.5+37	164 +65.5	9	0.4	6	R	B	
20	29.6	228 +33	220 +25	8	0.5	4	R	Y.W.	
21	34.6	239.5+ 5.5	231.5+17	4	0.4	1	R	Y	C
22	37.4	214.5+52.5	215.5+56.5	8	0.2	5	M	B	W
23	39.2	222 +19	223 +15.5	5	0.2	4.5	M	Y	W
24	43	213.5+46	191 +35.5	8	0.5	5.5	↗R	B	W
25	44.2	189 +54.5	181 +62.5	7	0.4	2	S	Y	
26	46.8	213.5+48.5	214 +53.5	8	0.2	5	S	B	W
27		195 +44	172 +46.5	7	0.5	4	M	Y	
			July 6, at Kyoto.						
1	8 50	217 +47.5	225 +55	7	0.3	3	↗R	Y	W
2	51	217 +49	213.5+52.5	8	0.2	5	M		W
3	51	214.5+48	214 +53	8	0.2	5.5	M		W
			July 11, at Kyoto.						
1	8 30	243.5−16	239.5−22	7	0.4	2	R	Y Curve	
2	9 6.4	181.5+54	169 +54.5	7	0.3	3.5	R	Brown	W
3	7.2	215 +47.5	213.5+53	7	0.2	5	↗R	B.W.	W
4	9.2	171.5+45.5	162.5+44	6	0.3	4	↗R	Y.W.	W
5	12.4	216.5+38	213 +36	6	0.2	4.5	↗R	Y	
6	13.2	217 +38.5	211 +39	6	0.2	5	↗R	Y	
7	15.5	201 +41	193.5+39	7	0.2	5	↗R	W	W
8	20.4	194 +42	188 +41.5	5	0.1	3	R	Y	W
9	22.2	216 +54	213 +52	7	0.2	5.5	↗R	W	
10	24.6	213 +47	213 +50	5	0.2	5	↗S	B	W
11	25.2	184.5+38.5	176 +38.5	5	0.2	4.5	↗R	B	W
12	27.2	238 +41	248 +39	7	0.3	4.5	↗R	W	W
13	30.8	215.5+42.5	219.5+45	5	0.2	5	↗S	B	W
14	33.2	213 +25	212.5+21	6	0.2	5	↗R	B	W
15	36.0	205 +50	200 +53	5	0.2	5	↗R	B	W
16	38.7	218 +42	224.5+40.5	6	0.3	5	↗S	B	W

No.	Time.	Appearance.	Dis-appearance.	Weight.	Dura-tion.	Magn.	Velo-city.	Color.	R.
	h. m.	o o	o o		s.				
17	9 40.6	191 +55	174 +61.5	7	0.3	5	R	B.W.	W
18	42.4	212.5+52.5	201 +55	8	0.3	2	M	Y	
19	44.2	211.5+47.5	207.5+50.5	6	0.2	5	M	B	W
20	45.2	248 +21	239 +27	8	0.4	2.5	R	Y	
			August 2, at Kyoto.						
1	8 58	182 +45.5	172.5+48.5	8	0.3	6	S	B	W
2	9 1	219 +27.5	216 +29.5	7	0.2	5.5	r R	B	W
3	4	221 +50	208.5+47.5	7	0.3	1	r S	Y & Blue	
4	7	252 +11	242.5+ 2	7	0.8	−3	r S	Y & Blue	
5	10	218.5+30	217 +32.5	6	0.1	6	r S	B	W
6	12	225.5+41.5	226.5+48	7	0.2	5.5	M	B	W
7	17	220 +28.5	218 +30	6	0.1	6	r S	B	W
8	17.1	222.5+27.5	226 +27.5	7	0.1	6	r S	R	D
9	21	221.5+20	216 +28	6	0.4	5	M	Purple	
10	25	209 +34.5	203.5+37	6	0.2	6	M	B	W
11	26	219 +28.5	216 +32	6	0.1	6	r S	B	W
12	30	281 +40	262.5+12	8	0.8	5	r R	R	
13	10 5	221 +28	222 +29	6	0.1	6	M	R	D
14	8	219.5+27.5	214.5+31	6	0.2	5.5	M	B.W.	W
			August 3, at Kyoto.						
1	8 34	173 +53	166 +45.5	8	0.5	4.5	S	R curved	
2	39	207.5+19.5	204.5+16.5	7	0.2	4.5	r R	C	D
3	40	270 +37	220.5+42.5	5	0.2	5	r R	C	D
4	43	231 +28.5	216 +51.5	8	0.5	5.5	M	B	
5	46	267 +58.5	265 +67	6	0.4	5	M	C	
6	47	220 +28	217.5+32	7	0.2	6	M	B	W
7	50	220 +29.5	215 +39.5	6	0.4	6	M	B	W
8	51	219.5+27	219 +28.5	6	0.1	5.5	M	C	
9	53	232.5+45.5	216 +37.5	7	0.5	5	M	B.W.	
10	57	221 + 9.5	201 +12	6	0.5	4	M	B	
11	9 5	210 +66	175 +60	6	0.5	5	r R	R	
12	9	225 +41	226 +50	7	0.4	5.5	r R	C	D
13	10	243 +25	240 +21.5	5	0.2	2.5	S	Y	
14	35	224 +23.5	211.5+25	7	0.4	5	r R	R.W.	
15	37	222.5+24	229.5+23.5	5	0.3	6	r S	W	
16	39	219 +27.5	216.5+29	6	0.2	6	r S		W
17	40	216 +25	211.5+25	6	0.2	6	r S	W	W
18	42	229.5+28.5	238 +30	7	0.3	5.5	r R	C	D
19	47	242.5+27.5	245.5+22.5	6	0.2	5	r R	W	
20	47	221 +25	224 +25	6	0.2	5.5	M		W
21	49	218.5+28	215 +28	6	0.2	5.5	r S		W

No.	Time.	Appearance.	Disappearance.	Weight.	Duration.	Magn.	Velocity.	Color.	R.
	h. m.	o o	o o		s.				
22	9 50	244 +61	232.5+69.5	8	0.5	4	M	Y.W.	D
23	53	218 +39	197.5+40	9	0.6	3	*r*S	Y	
24	55	217 +22	215 +17.5	6	0.3	5	*r*S	C	
25	59	223 +45	211 +39.5	6	0.4	4.5	*r*R	Y	
26	10 00	225 +48.5	227.5+55	7	0.3	5	*r*R	C	D
			August 4, at Kyoto.						
1	8 23	224 +26.5	229 +25.5	7	0.2	5	*r*R	R	D
2	26	220 +26	216.5+27.5	6	0.2	6	M	B	W
3	27	217.5+41	207.5+35.5	6	0.3	5	M	W	
4	30	218 +17	220 +11	5	0.3	5	M	Y	
5	31	209 +33.5	202.5+37.5	7	0.4	6	M		W
6	33	219 +27	214.5+29.5	6	0.2	6	M	B.W.	
7	33	198 +31.5	187.5+22	6	0.5	4	*r*S	R.Y.	W
8	36	220 +28.5	216.5+32.5	6	0.2	6	*r*S	W	W
9	39	221.5+23.5	217.5+24	6	0.2	6	S	B.W.	W
10	41	236 +17.5	238.5+ 6.5	7	0.4	4.5	*r*S	R	
11	44	263 −14	258.5−25	8	0.6	4	*r*S	C	
12	45	219 +26	205 +26.5	6	0.2	6	*r*S		W
13	46	219 +27.5	214.5+29.5	5	0.2	6	*r*S		W
14	49	250 +40.5	240 − 2	10	0.8	2	*r*R	Y Tail	
15	53	220.5+26.5	218 +30	6	0.2	6	*r*S	B.W.	W
16	53	220 +26	218 +28.5	6	0.1	6	*r*S	B.W.	W
17	55	162 +61	175 +49	7	0.5	2	M	Y Tail	
18	56	231.5+10	236 + 3.5	6	0.3	5	*r*R	R	D
19	59	245 +32.5	231.5+37.5	7	0.3	3	*r*R	Y	
20	9 1	214 +21.5	210.5+20	5	0.2	6	M	B.W.	W
21	4	213 + 9	196 +10	9	0.6	3	M	C	
			August 5, at Kyoto.						
1	8 27	217.5+37.5	216.5+41	6	0.1	6	M	B.W.	W
2	27.2	223 +38.5	223.5+42	6	0.2	6	M	B.W.	W
3	27.3	214.5+35	211.5+38	6	0.2	5	M	R	D
3	37	196.5+27.5	193.5+ 9.5	6	0.6	3.5	R	B	
5	41	262 +58	243 +62	7	0.4	3	*r*R	Y Tail	
6	54	215 +31	219 +41	7	0.3	3	R	B	
7	55	227.5+32	231.5+36	6	0.2	5	*r*R	R	D
8	9 00	227.5+34.5	230 +37.5	6	0.1	5	M	R	D
9	1	221.5+23.5	222 +26.5	5	0.2	6	S	B	W
10	13	226 +26	215 +40	8	0.3	5.5	R	W	
11	16	240 +31	231 +26	8	0.5	5	*r*S	C	
12	18	261.5+27.5	262.5+10.5	6	0.6	4.5	R	Green	
13	30	212 +40	225 +42.5	6	0.5	4	S	C	

No.	Time.	Appearance.	Dis-appearance.	Weight.	Dura-tion.	Magn.	Velo-city.	Color.	R.
	h. m.	o o	o o		s.				
14	9 35	222 +21	221 +17.5	5	0.2	6	S	B.W.	W
14	36	224 +19	207.5+21	6	0.5	2.5	rR	Y	
16	41	222.5+22.5	227.5+22	5	0.2	6	rS	B.W.	W
17	46	277.5+75	231 +71.5	7	0.4	4.5	R	R	
			August 7, at Kyoto.						
1		220.5+22	217.5+24.5			6			W
2		222 +19.5	220.5+16			6			W
3		222 +22.5	224 +25			6			W
4		223 +25	223 +27			6			W
5		224.5+26	226 +27.5			6			W
6		224 +21	227.5+19.5			6			W
7		227.5+23	231 +24.5			6			W
			August 8, at Kyoto.						
1		214 +27	209 +28.5			6			W
2		213 +25	208.5+26			6			W
3		218.5+21	215.5+19			6			W
4		220.5+20	218.5+18			6			W
5		225.5+24.5	225.5+27.5			6			W
6		232.5+19.5	237.5+16			6			W

§5. Statistics of the Results.

Daily numbers of meteors observed are arranged in the next table. The third column shows those meteors belonging to Winnecke's Comet, as proved by their pass on the daily chart, and at the same time by their appearances. (See §6).

TABLE II

Statistical Review.

Date.	Total Meteors.	Winnecke Meteors.	Interval.	Place.
1921 June 25	5	0	$13^{m.}$	Hachiman
26	32	14	60	,,
27	44	23	94	Fukuchiyama
28	40	29	60	Tottori
July 2	19	15	27	,,
3	51	26	84	Kyoto
4	48	13	53	,,
5	27	14	61	,,
6	3	3	—	,,
11	20	14	41	,,
August 2	14	8	37	,,
3	26	6	86	,,
4	21	10	44	,,
5	17	5	39	,,
7	7	7	20	,,
8	6	6	18	,,

Beside these observations, there are numbers of the Winnecke Meteors only counted by Nakamura, not recording their positions because of their too abundant apparitions, as in Table III:

TABLE III.

Counted Meteors belonging to the Winnecke.

Date.	Duration of Observations.	Total Number.	Place.
1921 July 3	$9^h\ 26^m$ ... $10^h\ 1^m$	153	Kyoto
5	9 27 ... 9 48	91	,,

From these tables, we see that the total meteors *seen and recorded* during the period from the end of June till the first decade of August is 380, of which those belonging to the Winnecke's Comet is 193, with, moreover, 244 meteors of the latter kind only *counted*. Table IV gives more detailed summary of the Winnecke meteors exclusively.

From the above results, we see that the climax of the showery displays was between the 29th and the 30th of June, which is reasonably justified by the elements of the orbit of the comet.

TABLE IV.

Date.	Radiant Point.	Diffusion.	Hourly Number.	Weather.	Mean Magnitude
1921					
June 26	211°.5 +52°	2°	14	Cloudy	5.4
27	210 +50.5	2.5	15	Cloudy	5.0
28	212.5 +49	5.5	29	Cloudy	5.1
July 1	212 +48	5	33	Cloudy	3.5
3	213 +47	4	19	Fine	4.8
4	213 +46.5	1.5	15	Cloudy	4.8
5	213.5 +46	2	14	Very Fine	4.8
6	214 +45.5			Cloudy	4.5
11	215 +42	2	20	Fine; Moon	5.0
Aug. 2	222.5 +25		11	Fine	5.8
3	222 +25		4	Very Fine	5.8
4	223 +24		14	Fine	6.0
5	222.5 +22.5		8	Cloudy	6.0
7	222.5 +21		21	Fine; Moon	6.0
8	225 +22.5		20	Fine; Moon	6.0

But the estimations of the hourly numbers is naturally influenced by the weather conditions, and that especially in this kind of very faint meteors. Consequently, the observations with special attention to the meteoric displays *sacrificing labours of recording* resulted in the following figures for hourly numbers on the nights of July the 3rd and 5th, as deduced from the data of Table III:

July 3, hourly number = 262,

„ 5, „ „ = 260.

The march of the radiant point during the period of observations is well shown. The total range actually observed is about 30° in six weeks. These radiant points were estimated in our maps used in the fields. Here we see that the radiant points are diffused to some extent, amounting more than five degrees in some instances. But, while counting only were made on July 3rd and 5th., some meteors

radiated from within 1.5 degrees of the radiant point, rarely, indeed, within half a degree.

§6. Characteristics of Apparitions.

Trail. The general feature of this kind of meteors belonging to Winnecke's Comet was its ash-like appearance, by which we could easily distinguish the present objects from other meteors. This is quite in accordance with Mr. Denning's description:

> "..... The meteors were rather slow, and the brighter ones left trains. In some cases the nuclei were resolved into streams of luminous ashes and presented essentially different aspects from the usual stellar meteors." (Observatory, Vol. 39, 356.)

These impressions made one of us (K.N.) recall a similar kind of apparitions seen on the night of June 23 through small gaps of clouds while watching the sky at Kyoto. He was little expecting at that time to see the Winnecke, because the date seemed too early for them. But, lately, by accumulated experiences, he has become assured in the belief that those that were seen that night were really meteors of the present shower. If it had been clear weather, his discovery must have been, thus, at least, three days earlier. Average velocities of the recorded meteors were found rather to be rapid. But as they were generally short, and the condition of appearance was peculiar, it was pretty hard to know their real velocities.

The curious fact was that the meteors were numerous in two regions of position angles, that is in 150°–180° and 330°–369°. These directions are opposite to each other. This may be probably due to some psychological effects, affected by the neighbouring configurations of bright stars.

Magnitude and Colour. Meteors of this shower were generally very faint; the brightest being of the 2nd magnitude, and that rarely. The recorded meteors were generally of the 5.5th magnitude and brighter. See the following table.

Table V.

Date	*Magnitudes.*								
	2	2.5	3	3.5	4	4.5	5	5.5	6
June 26	—	—	—	—	—	1	3	8	2
27	—	—	—	1	1	4	9	8	—
28	1	—	—	—	—	2	12	10	1

July	1	1	1	5	3	3	1	—	1	—
	3	—	1	2	—	4	1	9	7	2
	4	—	—	—	—	1	5	6	1	—
	5	—	—	1	—	1	2	6	4	—
	6	—	—	1	—	—	—	1	1	—
	11	—	—	1	1	1	—	2	9	—
Aug.	2	—	—	—	—	—	—	—	3	5
	3	—	—	—	—	—	—	—	2	4
	4	—	—	—	—	—	—	—	—	10
	5	—	—	—	—	—	—	—	—	5
	7	—	—	—	—	—	—	—	—	7
	8	—	—	—	—	—	—	—	—	—

The colour of the present meteors were generally of three kinds:— blue, bluish-white and white. But some fine meteors were yellow. The following table shows the distribution:

TABLE VI.

Colour-frequency of Winnecke meteors.

Date.	B	BW	W	Y	RW	YB	Brown
June 26	4	1	2	—	—	—	—
27	2	2	—	—	—	—	—
28	1	10	11	—	—	—	—
July 1	—	—	—	—	—	—	—
3	—	2	9	2	2	—	—
4	4	2	4	—	—	—	—
5	7	—	1	2	—	1	—
6	—	—	—	1	—	—	1
11	6	2	1	2	—	—	—
Aug. 2	7	1	—	—	—	—	—
3	2	—	1	—	—	—	—
4	1	5	1	—	—	—	—
5	1	4	—	—	—	—	—
7	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—
Sum	35	29	30	7	2	1	1
%	33	28	29	7	2	1	1

§7. Other Showers of Meteors.

In the observed meteors of Table I, there are many that do not belong to the Winnecke, of which the following four showers are found to have each its own radiant points:

Table VII.

Date.	Radiant Point.	Designation.	Number.
June 25–28	212°.5 + 19°.5	α Bootids	9
June 26–July 3	242° + 60°	η Draconids	6
July 3– 5	251 − 4	Ophiuchids	8
Aug. 2– 5	221 + 27	ε Bootids	13

The last one was very near the Winnecke in position. But we could detect the differential character by the peculiarities of the latter: for, these Bootids were very distinct with their red colour and with streaks of the usual stellar form. This shower showed a moving radiant point, thus:

August 2, 221° + 28°
„ 3, 220.5 + 27
„ 5, 221 + 25.5

§7. Summary.

Although much perturbed by peculiar weather conditions, owing to the annual rainy season, the expected returns of the meteoric displays probably belonging to the Pons-Winnecke's Comet were confirmed by actual discoveries and observations. They were generally very faint, of ash-like appearance, and left trains in the brighter ones. The progression of the radiant points was followed over some six weeks, which were pretty diffuse. Of colours, blue to white were predominating.

Many thanks are due to the following gentlemen: Mr. E. Yoshida in Hachiman, Mr. Y. Iwamori in Fukuchiyama, and Mr. T. Ide in Tottori, through whose kindness the above observations were safely made.

I. Yamamoto.
K. Nakamura.

Kyoto University Observatory,
September 1, 1921.

Some Considerations on the Indeterminate Series

By

Toshizô Matsumoto.

(Received January 11, 1922.)

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1. The present note is the continuation of the previous paper on some properties of the analytic elements on and outside their circles of convergence; but this paper is not necessarily related to the analytic continuations. In the previous paper I proved that a power-series (real) which is convergent for $0 \leqq x < 1$ divides the interval $0 \leqq x$, if possible, into three parts: 1°. It is convergent for $0 \leqq x < 1$; 2°. infinite for $1 < x < x_0$ 3°. indeterminate for $x_0 < x$.[1] Now it is our problem whether x_0 may be unity, *i.e.* whether it gives a series which is convergent for $0 \leqq x < 1$, and indeterminte for $1 < x$, but at $x = 1$, it is infinite. To verify this, we shall construct in the following a series which is really so.

2. Let the required series be

$$f(x) = a_0 + a_1 x + \ldots\ldots + a_n x^n + \ldots\ldots,$$

and put

$$S_n \equiv a_0 + a_1 + \ldots\ldots + a_n,$$
$$S'_n \equiv a_1 + 2a_2 + \ldots\ldots + na_n.$$

If S'_n be positive for any n, then since it is the differential quotient of the polynomial $a_0 + a_1 x + \ldots\ldots + a_n x^n$ at $x = 1$, this polynomial must be increasing at $x = 1$. It would be the same for $f(x)$

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1 Mem., Coll. Sci., Kyoto, **4,400,** (1921).

or $f(x)$ could not indeterminate in the neighbourhood of $x>1$. We have therefore to determine $a_0, a_1, \ldots\ldots, a_n, \ldots\ldots$ such that

$$\lim_{n\to\infty} S_n=\infty, \quad S'_n<0,$$

for certain increasing values of n. Since

$$S'_n=(S_1-S_0)+2(S_2-S_1)+\ldots\ldots+n(S_n-S_{n-1}),$$

we have

$$\frac{S'_n}{n+1}=S_n-\frac{S_0+S_1+\ldots\ldots+S_n}{n+1}. \tag{1}$$

Referring to this identity, take

$$\begin{gathered} S_0=0, \quad S_1=1, \quad S_2=2, \\ S_{1+2}=1, \quad S_4=4, \quad S_5=5, \\ S_{1+2+3}=2, \quad S_7=7, \quad S_8=8, \quad S_9=9, \\ \ldots\ldots\ldots\ldots \end{gathered}$$

In general, if $n=1+2+3+\ldots\ldots+m=\frac{m(m+1)}{2}$,

$$S_n=m-1, \tag{2}$$

or else

$$S_n=n. \tag{3}$$

Then for $n=\frac{m(m+1)}{2}$, we have by easy calculations

$$S_0+S_1+\ldots\ldots+S_n=\frac{m^4}{8}+\frac{m^3}{12}+\frac{3m^2}{8}-\frac{7m}{12}+1. \tag{4}$$

Since for this value of n, $S_n=m-1$, (1) shows that S'_n tends to $-\infty$ when m tends to ∞. For the coefficients of the series we have

$$\left.\begin{aligned} a_0&=0, \\ a_n=S_n-S_{n-1}&=-\frac{m(m-1)}{2}, \text{ for } n=\frac{m(m+1)}{2}, \\ &=\frac{m^2-m+4}{2}; \text{ or } n=\frac{m(m+1)}{2}+1. \end{aligned}\right\} \tag{5}$$

For the other cases, we have

$$a_n = 1. \tag{6}$$

Hence

$$\overline{\lim_{n\to\infty}} \sqrt[n]{|\, a_n \,|} = 1.$$

Now the series

$$f(x) = x + x^2 - x^3 + 3x^4 + x^5 - 3x^6 + 5x^7 + \quad .. \tag{7}$$

is what is required. For at first we have

$$\lim_{n\to\infty} S_n = f(1) = \infty .$$

Next we can easily verify that the series is indeterminate for any $x > 1$. To show that the series becomes positive by certain modes of summation, observe that for all $n = \frac{m(m+1)}{2}$, by (5),

$$a_n x^n + a_{n+1} x^{n+1} = \frac{1}{2} x^{\frac{m(m+1)}{2}} \{ -m(m-1) + (m^2 - m + 4)x \}$$

$$= \frac{1}{2} x^{\frac{m(m+1)}{2}} \{ m(m-1)(x-1) + 4x \} > 0$$

for any $x > 1$. The other terms of $f(x)$ are all positive. Therefore if we take in $f(x)$ the above sum as one term, then $f(x) \to \infty$, for any $x > 1$. To see that $f(x)$ becomes negative by certain modes of summation, consider the sums for all $n = \frac{m(m+1)}{2} + 1$, by (5) and (6),

$$T_n(x) \equiv a_n x^n + a_{n+1} x^{n+1} + \ \ldots\ldots + a_{\frac{(m+1)(m+2)}{2}} x^{\frac{(m+1)(m+2)}{2}}$$

$$= x^n \left\{ \frac{m^2 - m + 4}{2} + x + \ldots\ldots + x^{m-1} - \frac{m(m+1)}{2} x^m \right\}.$$

The terms within the blackets are equal to

$$\frac{m^2 - m + 2}{2} + \frac{x^m - 1}{x - 1} - \frac{m(m+1)}{2} x^m,$$

which is always negative for any $x > 1$, provided m is sufficiently great. Thus we can find N, such that $n \geqq N$ derives for each $x > 1$,

$$T_n(x) < 0.$$

By aid of this inequality, we sum up $f(x)$ in the following mode

$$a_0 + a_1 x + \ldots\ldots + a_{N-1} x^{N-1} + T_N(x) + T_{N+1}(x) + \ldots\ldots$$

Since $T_N(x)$, $T_{N+1}(x)$, ... are all negative, this series can not be ∞, moreover tends to $-\infty$. Thus *our series* $f(x)$ *is convergent for* $0 \leqq x < 1$, ∞ *at* $x=1$, *and indeterminate for* $x>1$.

3. We shall consider in the following sections some relations between $f(x)$ and $f'(x)$ at $x=1$, where as usual in the series

$$f(x) = a_0 + a_1 x + \ldots\ldots + a_n x^n + \ldots\ldots,$$

a_0, a_1, ..., a_n, ... are all real and $f(x)$ has the radius of convergence unity.

4. It is clear that when $f(1)$ is convergent, $f'(1)$ may converge, or be divergent. As a special case, we notice that Mr. Vivanti has given a series which is convergent with all its derivatives at $x=1$.[1] When $f(1)$ is ∞, $f'(1)$ may be ∞ or indeterminate. The former case is clear, while the latter is given by the series (7) found in the previous section. As we have said, for $n = \frac{m(m+1)}{2}$, $\lim_{m\to\infty} S'_n = -\infty$. On the other hand by (1), (3), (4), we have

$$\frac{S'_{n+1}}{n+2} = \left\{ \frac{m(m+1)}{2} + 1 \right\} - \frac{\frac{m^4}{8} + \ldots\ldots}{\frac{m(m+1)}{2} + 2},$$

the dotted part stands for the lower degrees of m. Consequently

$$\lim_{m\to\infty} S'_{n+1} = \infty.$$

So that our series $f'(x)$ is indeterminate at $x=1$.

5. When $f(1)$ is indeterminate, $f'(1)$ is so also. We shall prove a more general proposition. Let us write

$$S \equiv a_0 + a_1 + \ldots\ldots + a_n + \ldots\ldots,$$
$$S_n \equiv a_0 + a_1 + \ldots\ldots + a_n,$$
$$S' \equiv \omega_0 a_0 + \omega_1 a_1 + \ldots\ldots + \omega_n a_n + \ldots\ldots,$$
$$S'_n \equiv \omega_0 a_0 + \omega_1 a_1 + \ldots\ldots + \omega_n a_n,$$

[1] Vivanti-Gutmer, Theorie der eindeutigen analytischen Funktionen, (1906), p. 413.

where $\omega_0, \omega_1, \ldots\ldots, \omega_n$ form a positive increasing sequence.

When S is indeterminate, S' must also be indeterminate. For, writing

$$\theta_i = \frac{1}{\omega_i}, \qquad i = 0, 1, \ldots, n, \ldots,$$

we have

$$\theta_0 \geq \theta_1 \geq \ldots\ldots \geq \theta_n \geq \ldots\ldots,$$
$$S_n = (\theta_0 - \theta_1)S'_0 + (\theta_1 - \theta_2)S'_1 + \ldots\ldots + (\theta_{n-1} - \theta_n)S'_{n-1} + \theta_n S'_n. \quad (8)$$

If $\lim_{n\to\infty} S'_n = \infty$, since for any integer $i \geq 0$, $\theta_i - \theta_{i+1} \geq 0$, $\theta_i > 0$, the right-hand side of (8) must increase with n; hence S must be convergent or ∞. It is contrary to the assumption. If for any $n > N$, $|S'_n| < \varepsilon$, by Abel's inequality

$$|a_n + a_{n+1} + \ldots\ldots + a_{n+p}| \leq 2\varepsilon\theta_n.$$

Therefore S becomes convergent. It is also absurd. *Q.E.D.*

Taking $\omega_i = i$,[1] *if a power-series* $f(x) = a_0 + a_1 x + \ldots\ldots a_n x^n + \ldots\ldots$ *be indeterminate at* $x = 1$, *then* $f'(1)$ *must also be indeterminate and hence it can not be limited in both ways.*

As an example, since

$$\sin\alpha + \sin 2\alpha + \ldots\ldots + \sin n\alpha + \ldots\ldots$$

is indeterminate, the indeterminate series

$$\sin\alpha + 2\sin 2\alpha + \ldots\ldots + n\sin n\alpha + \ldots\ldots$$

can not be limited in both ways, since the power-series

$$x\sin\alpha + x^2\sin 2\alpha + \ldots\ldots + x^n \sin n\alpha + \ldots\ldots$$

has the convergent-radius unity.

6. The relations of the oscillation-abscissa of S' with respect to that of S is very important. But it is in general extremely difficult to find. We shall merely prove that if $\frac{S'_n}{n}$ is limited in a positive way (*i.e.*, it is less than a number G for any n), then $\frac{S_n}{\log n}$ is also limited in a positive way, where

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[1] When $\omega_0 = 0$, we consider $S - a_0$ instead of S.

$$S_n = a_0 + a_1 + \ldots\ldots a_n,$$
$$S'_n = \quad a_1 + \ldots\ldots + n a_n.$$

For this, we use the identity (8) for $\omega_i = i$:

$$S_n - a_0 = \left(1 - \frac{1}{2}\right) S'_1 + \ldots\ldots + \left(\frac{1}{n-1} - \frac{1}{n}\right) S'_{n-1} + \frac{1}{n} S'_n$$

Since $\frac{S'_i}{i} < G$, $i = 1, 2, \ldots, n, \ldots$, we have

$$S_n - a_0 < \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}\right) G.$$

We know that

$$\lim_{n\to\infty} \frac{1 + \frac{1}{2} + \ldots\ldots + \frac{1}{n}}{\log n} = 1.$$

Therefore we can find a number G' such that for all n

$$\frac{S_n}{\log n} < G'. \qquad \textit{Q.E.D.}$$

To obtain deeper relations, we have probably to use more transcendental analytical arms or add another condition. Tauber's problem is an allied one. He proved that if $S'_n = o(n)$, $\lim_{x\to 1} f(x) = o$, then $S = o$. Messrs. Hardy and Littlewood extended this theorem.[1] They also proved another, in which instead of $f(x)$, Borel's generalised sum of S was used. It was extended by Mr. Fujiwara.[2]

7. When $f(x) = a_0 + a_1 x + \ldots\ldots + a_n x^n + \ldots\ldots$ may be continued over $x = 1$, may $f_\omega(x) \equiv \omega_0 a_0 + \omega_1 a_1 x + \ldots\ldots + \omega_n a_n x^n + \ldots\ldots$ also be continued over $x = 1$? $\omega_0, \omega_1, \ldots, \omega_n, \ldots$ are the same quantities defined above. In the special case where $\omega_0 = o$, $\omega_n = n$, *i.e.*, in the case of analytic functions, the answer is clearly affirmative. But in general it is not the case and the problem is related to Hadamard's theorem. For if $x = -1$ be a singular point of $f(x)$ and of the function defined by

$$\Omega(x) \equiv \omega_0 + \omega_1 x + \ldots\ldots + \omega_n x^n + \ldots\ldots,$$

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[1] Landau, Darstellung und Begründung einiger neuerer Ergebnisse der Funktionentheorie, (1916; J. Springer).

[2] Tôhoku Math. J., **17**, Nos. 3, 4, (1920).

and even when $f(x)$ may be continued over $x=1$, the point $x=1$ is a singular point of the analytic function defined by $f_\omega(x)$. Therefore given a series $f(x)$ which may be continued over $x=1$, we may divide all the positive increasing sequences into two parts in one of which all the sequences for which $f_\omega(x)$ may be contined over $x=1$, are classified; while in the other, all the sequences for which $f_\omega(x)$ may not be continued. Then if $f_\omega(x)$ and $f_{\omega'}(x)$, corresponding to the sequences

$$\omega_0, \omega_1, \ldots\ldots, \omega_n, \ldots\ldots,$$
$$\omega'_0, \omega'_1, \ldots\ldots, \omega'_n, \ldots\ldots$$

respectively may be continued, the series $f_{\omega+\omega'}(x)$ corresponding to the sequence $\omega_0+\omega_0, \omega_1+\omega'_1, \ldots\ldots, \omega_n+\omega'_n, \ldots\ldots$ may also be continued. On the other hand consider a sequence $\varepsilon_0, \varepsilon_1, \ldots\ldots, \varepsilon_n, \ldots\ldots$, tending to zero. Even when $f_\omega(x)$ may be continued, yet it may occur that $f_{\omega+\varepsilon}(x)$ may not be continued. For if otherwise, $f_\varepsilon(x)$ must be continued over $x=1$. This is in general untrue.

We will not go further since probably such considerations can not throw light on the indeterminate series, though these considerations themselves are interesting. Here the researches of Borel and of Pincherle should be referred to.[1]

8. Given a limited function $F(x)$ defined in an interval, Darboux established the so-called *intégrales par excès* and *par défaut*. To these integrals, it seems to correspond the superior and inferior limits of the indeterminate series which are limited.

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1 See *e.g.*, Montel, Leçons sur les séries de polynomes, (1910), p. 37.

Constitution of Polysaccharides, I.
Xylan and Its Acetyl Derivatives, I.

By

Shigeru Komatsu and Kozo Kashima.

(Received January 19, 1922.)

1. *Xylan.*

The xylan used in the present investigation was obtained from wheat straws and corn cobs grown in Kyoto, by following generally the method of Allen and Tollens.[1]

150 grm. air dried wheat-straws were treated in the cold for twenty-four hours with ammonia water (120 c.c. of conc. ammonia in 850 c.c. water); the ammoniacal solution is then removed by decantation and by pressing. The material after washing with water, was immersed in 300 c.c. of 7 % sodium hydroxide solution for forty-eight hours. The extract was separated from insoluble matter by filtration through a silk cloth. To the clear brown-coloured solution, an equal volume of 98 % alcohol was added with constant stirring to precipitate the xylan; the mohter liquor was siphoned off and the precipitate washed with dilute alcohol and dissolved again in 7 % sodium hydroxide solution and reprecipitated with alcohol. The precipitate separated from the mother liquor, was treated with dilute hydrochloric acid and dilute alcohol until all hydrochloric acid was removed, washed with ether three times and dried on paraffin and sulphuric acid in vacuo. The yield was 25 grm. (16.7 %).

Xylan thus prepared is a light-yellow amorphous powder, easily soluble in alkaline solution and showed the specific rotation $[\alpha]_D = -78°$

Lieb. Ann. d. Chem., **260**, 290 (1890).

in 2·5 % sodium hydroxide solution, but is insoluble in water and alcohol. Analysis of the substance gave the following results:

I. 0·3155 grm. substance gave 0·5204 grm. CO_2, 0·1643 grm. H_2O and 0·0025 grm. ash.

II. 0·1355 grm. substance gave 0·2213 grm. CO_2, 0·0817 grm. H_2O and 0·0011 grm. ash.

		Found. (ash-free)	
	Calc. for $C_5H_8O_4$	I	II
Carbon	45·43	45·35	45·01
Hydrogen	6·16	5·83	6·81
Ash	—	0·79	0·79

On heating xylan with water at 120–130° for three hours in a sealed tube, no change could be observed, however, at 145° for three hours it yields a trace of xylose by hydrolysis. The hydrolysis of xylan was also in a slight degree observed when heated with phosphoric acid at 145° for three hours. On the other hand, 0·496 grm. of the xylan upon heating with 1·5% hydrochloric acid on a water-bath for two hours, is quickly hydrolysed to laevo-xylose, indicating by Fehling's solution 0·2418 grm. xylose (48·6%) present in the reaction product. In another experiment, 10·4468 grm. of the xylan heated with 4 % sulphuric acid on a water bath for two hours, yielded 62·3% xylose and 3% fumic acid[1] which separated and was confirmed by its analysis:

0·0986 grm. substance gave 0·2091 grm. CO_2, 0·0611 grm. H_2O and 0·003 grm. ash.

	Found. (ash-free)
Carbon	59·64
Hydrogen	7·09
Ash	3·13

3·8 grams of the laevo-xylose (36·38%) were isolated in a pure crystalline state from the above reaction product, with 1·955 grm. xylose remaining in the solution.

Xylan prepared from corn cobs by the same treatment as shown in the above case, is a slight yellow-coloured amorphous powder and showed the specific rotation $[\alpha]_D = -80°$ in 2·5% sodium hydroxide solution.

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[1] S. Odén, Die Huminsäuren, 1919, s. 100.

It gave the following analytical results:

C 45·22 %, H 5·88 % and ash 1·28 %.

It is a noteworthy fact that xylan prepared from corn cobs hardly yields an acetyl derivative by the action of acetic anhydride and acetyl chloride with or without catalytic agents, whereas xylan from wheat straws yields easily acetyl compounds.

2. *Acetyl Xylans.*

As we saw in an acetylation of polysaccharides such as starch and cellulose[1], the velocity of the acetylation of xylan was observed to be greatly influenced by the presence of a catalytic agent.

Monoacetyl Xylan.[2]

A mixture of 2 grm. of the wheat xylan 25 grm. acetyl chloride was heated in a pressure bottle on a water bath; all of the mass passed into a light brown solution in five minutes. On adding 70 c.c. absolute alcohol to the solution, a white voluminous substance was precipitated which separated from the mother liquor by filtration, dried on air. To purify, the product was dissolved in chloroform, filtered, and the filtrate poured into 50 c.c. petroleum ether to precipitate the acetyl compound, which after being separated from the solution was dried in vacuo. The yield was 1 grm. (40%).

It is a light yellow amorphous powder and is soluble in chloroform and an alkaline solution but insoluble in ether and alcohol.

0·3162 grm. of the acetyl compound were digested with 74·28 c.c. $\frac{N}{2}$ H_2SO_4 for ten hours, filtered and washed with water. The filtrate and washing made up 100 c.c. with water. For the neutralization of 10 c.c. of the solution requires of 7 c.c. $\frac{N}{2}$ NaOH.; whence acetyl value 25·48; theory requires 24·72%.

For the preparation of monoacetyl xylan, the following method is very convenient and affords a fruitful result, and was formerly employed in the preparation of acetyl cellulose by W. L. Barnett.[3]

10 grm. xylan suspended in a mixture of 60 grm. glacial acetic acid and 85 grm. acetic anhydride, were mixed with 40 grm. of 2·45%

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1 J. Böeseken, J. C. van den Berg, & A. H. Kerstjens, Rec. trav. chim., **35**, 320 (1916).

2 R. Bader, J. Chem. Soc., **70**, 1, 335 (1896).

3 J. Soc. Chem. Indust., **40**, 8 (1921).

chlorine containing glacial acetic acid and 18·8 grm. of 4·66% sulphur dioxide containing glacial acetic acid. The mixture was shaken for 30 minutes and the xylan passed into solution with a slight evolution of heat. The solution was poured into cold water and the precipitate purified was dried in vacuo. The yield was 9 grm. (70%).

On analysis, it gave the following results:

I. 0·1252 grm. substance gave 0·2187 grm. CO_2, 0·0679 grm. H_2O and 0·0008 grm. ash.

II. 0·0719 grm. substance gave 0·0923 grm. $BaSO_4$ on hydroylsis by the method of Herzig.

		Found (ash-free)	
	Calc. for $C_5H_7O_4(CH_3CO)$	I	II
Carbon	48·26	47·95	—
Hydrogen	5·79	6·06	—
Acetyl	24·72	—	23·64
Ash	—	0·64	—

0·2186 grm. of the substance dissolved in chloroform to make up 10 c.c. $\alpha_D^{21} = -4°·25'$ in one dm. tube; whence $[\alpha]_D^{21} = -202°·6'$.

The molecular weight of the substance was determined in the chloroform solution by the ebullioscopic method:

			Mol. wt.	
Solvent	Substance.	△	Found.	Calc. for $(C_7H_{10}O_5)_2$
31·6543 grm.	0·0580 grm.	0·02°	337·5	
,,	0·2761 grm.	0·09°	357·0	
		Mean	347·2	348

Diacetyl Xylan.[1]

In the course of the acetylation of xylan with acetyl chloride, when pyridine was used as catalyser, diacetyl xylan was formed. 3·5 grm. xylan, 70 grm. acetyl chloride and 3 grm. pyridine in 15 c.c. glacial acetic acid were heated on a water bath for five minutes. The light-brown-coloured solution thus obtained, was poured into 100 c.c. absolute alcohol. The precipitate was washed with alcohol and dried in air and dissolved in chloroform and reprecipitated with petroleum

[1] R. Bader, J. Chem. Soc, **70**, 1335 (1896)

ether; the same process was twice repeated and then the substance dried in vacuo.

The yield was 2 grm. (35 %).

On analysis, it gave the following results:

0·1881 grm. substance gave 0·3413 grm. CO_2, 0·0946 grm. H_2O and 0·0007 grm. ash.

	Calc. for $C_5H_6O_4(CH_3CO)_2$	Found (ash-free)
Carbon	49·97	49·68
Hydrogen	5·56	5·60
Ash	—	0·37

0·1986 grm. substance gave 0·2075 grm. $BaSO_4$ on hydrolysis; whence acetyl value 38·57% theory requires 39·79%.

This compound was obtained from xylan and acetic anhydride and an excess of chlorine and sulphur dioxide used as a catalyser. The mixture 10 grm. xylan in 55 grm. glacial acetic acid, 100 grm. of 2·45% chlorine-glacial acetic acid and 50 grm. of 4·66% sulphur dioxide-glacial acetic acid, shaken in a bottle for 30 minutes; all passed into solution which was then poured into water. The acetyl compound separated from the solution, dissolved in chloroform and reprecipitated by adding petroleum ether. The yield was 9 grm. (55%).

I. 0·1681 grm. substance gave 0·3042 grm. CO_2, 0·0905 grm. H_2O and 0·0007 grm. ash.

II. 0·1970 grm. substance gave 0·2196 grm. $BaSO_4$ on hydrolysis by the method of Herzig.

		Found (ash-free)	
	Calc. for $C_5H_6O_4(CH_3CO)_2$	I	II
Carbon	49·97	49·58	—
Hydrogen	5·56	5·97	—
Acetyl	39·79	—	41·06
Ash	—	0·42	—

It was also prepared from xylan by the action of acetic anhydride and zinc chloride; 2 grm. xylan mixed with 15 grm. acetic anhydride and 2 grm. zinc chloride, and heated on water bath for five minutes. The reaction product was poured into water and the precipitated diacetyl xylan was purified with chloroform and petroleum ether as usual. The yield was 1·7 grm. (52%).

0·1388 grm. substance gave 0·2526 grm. CO_2, 0·0701 grm. H_2O and 0·0005 grm. ash.

	Calc. for $C_5H_6O_4$ $(C_2H_3O)_2$	Found (ash-free)
Carbon	49·97	49·97
Hydrogen	5·56	5·63
Ash	—	0·36

The same diacetyl compound is obtained when monoacetyl xylan is treated with acetic anhydride in the presence of chlorine and sulphur dioxide.

Diacetyl xylan is a light yellow amorphous powder, and is soluble in chloroform but insoluble in ether and alcohol. The specific rotation of the compound in chloroform solution was observed and the following result was obtained:

0·2170 grm. substance dissolved in chloroform make up to 10 c.c. gave $\alpha_D^{20·5} = -2°·57'$ in 1 dm. tube; whence $[\alpha]_D^{20·5} = -135°·9'$.

The substance standing in contact with sodium hydroxide solution passed gradually into solution.

3. *Hydrolysis of Xylan and Acetyl Xylans.*

Certain quantities of xylan and acetyl compounds were mixed with $\frac{N}{5}$ $C_6H_5SO_3H$ solution, and the mixture kept in a boiling water bath, and the quantity of reducing substance formed by the hydrolysis was determined by the aid of Fehling's solution. The constant of the reaction velocity was calculated by the formula of a monomolecular reaction as in the case of the hydrolysis of polysaccharides by E. Hildt.[1]

I. Xylan from wheat-straw.

Substance (in grm.)	T (minute)	Cu_2O (grm.)	x (grm. xylose)	$K = \frac{2·3}{T} \log \frac{a}{a-x}$
0·1378	20	0·1116	0·0521	0·024
0·1361	40	0·1262	0·0594	0·014
0·1267	60	0·1344	0·0636	0·012
0·1221	80	0·1560	0·0747	0·012
0·1303	100	0·1680	0·0810	0·010
			Mean	0·014

[1] C. R., **170**, 1505 (1920).

II. Monoacetyl xylan.

Substance (in grm.)	T (minute)	Cu_2O (grm.)	x (grm. xylose)	$K=\frac{2\cdot3}{T}\log\frac{a}{a-x}$
0·1905	90	0·0284	0·0126	0·0008
0·1446	180	0·0576	0·0261	0·0011
0·1442	270	0·0838	0·0384	0·0011
0·1238	360	0·1010	0·0468	0·0013
0·1208	450	0·1000	0·0464	0·0011
			Mean	0·0011

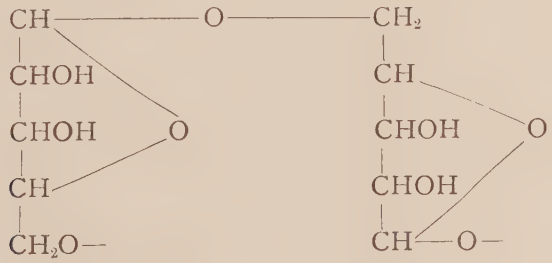
III. Diacetyl xylan.

Substance (in grm.)	T (minute)	Cu_2O (grm.)	x (grm. xylose)	$K=\frac{2\cdot3}{T}\log\frac{a}{a-x}$
0·1269	90	—	—	—
0·1358	180	0·0054	0·0028	0·00012
0·1238	300	0 0140	0·0062	0·00017
0·1216	425	0·0240	0·0106	0·00027
0·1386	530	0·0550	0·0249	0·00037

From the experimental data, we have learned that the time in which 18% of mono- and diacetyl xylans have been hydrolysed, requires 18 and 530 minutes respectively, whereas the same quantity of xylan can be hydrolysed in a few minutes. The discrepancy in the constants of reaction velocity of diacetyl xylan calculated and observed, may be explained by assuming that autocatalysis takes place during the hydrolysis.

4. Constitution.

Xylan is a non-reducing polysaccharide which yields laevo-xylose on hydrolysis with mineral acids such as hydrochloric and sulphuric acids. Consequently, it is regarded as a polyose in which a reducing group of one xylose molecule is jointed with the hydroxyl group of another molecule as indicated below:



The above formula contains two free hydroxyl groups which indicate xylan by the acetylation yields monoacetyl and diacetyl compounds.

The exact constitution of xylan, however, is still a matter of doubt and will be studied and reported in a future article.

Photochemische Untersuchung über die Einwirkung von Ferrisalzen auf Jodide.

(Eine neue umkehrbare photochemische Reaktion)

Von

Nobuji Sasaki.

(Eingegangen am 29. Januar, 1922.)

Unter den zahlreichen photochemischen Vorgängen lassen sich nur eine ganz beschränkte Anzahl umkehrbarer photochemischer Reaktionen in Bezug auf den gasförmigen, flüssigen, ebensowie auf den festen Zustand erwähnen. Bisher hat man noch keinen Fall gefunden, wo die wässerige Salzlösung etwa eine wichtige Rolle gespielt hätte. Doch schon aus dem Grunde, dass bei wässerigen Salzlösungen die thermodynamischen, elektrochemischen und viele andere wichtigen Beziehungen deutlicher und genauer zur Verfügung stehen, ist es sehr wünschenswert, auch auf diesem Gebiete ein solches Beispiel haben zu können.

Die Umsetzung zwischen Ferrisalzen und Jodiden in wässeriger Lösung, die die Jodabscheidung zur Folge hat, wird oft zum analytischen Zwecke angewandt, und ihre quantitativen Beziehungen wurden schon von Seubert[1] untersucht. Nach ihm verläuft die Reaktion mit einer mässigen Gechwindigkeit bis zu einem Gleichgewichtszustande, welcher seinerseits bei der Umsetzung zwischen Ferrosalzen und freiem Jod erreicht werden kann. Also sind die Reaktionen im Sinne der Gleichung[2]

$$Fe''' + J' \rightleftarrows Fe'' + J$$

aufzufassen. Durch Temperaturerhöhung wird die Reaktionsgeschwindigkeit vergrössert und das Gleichgewicht zugunsten der Jodabscheidung

1 Zts. f. anorg. Chem. **5**, 334, 339; **7**, 137, (1894).

2 Küster, ebenda, **11**, 165, (1896).

verschoben, vorausgesetzt dass die Hydrolyse von Eisensalzen durch Säurezusatz vermieden würde. Von ihm ist aber noch nicht darauf hingedeutet worden, dass das Licht auf diese Reaktion einen Einfluss auszuüben vermochte, und bis jetzt ist noch keine diesbezügliche Mitteilung erschienen. Trotzdem kann man aber aus der grossen Rolle, welche diese Reaktionsteilnehmer in der Photochemie spielen, mit Recht vermuten, dass auch hier irgend eine photochemische Wirkung zum Vorschein kommen würde, und hierauf beziehen sich die unten mitzuteilenden Versuche.

Qualitative Versuche.

Es wurde das Gemisch von Eisenalaun- und von Jodkaliumlösung frisch in zwei Probierröhren dargestellt und die eine dem Sonnenlicht ausgesetzt, die andere aber im Dunkeln gelassen. Die beiden Flüssigkeiten wurden durch allmähliche Jodabscheidung langsam brauner; dazwischen wurde aber sofort ein Unterschied der Nuance beobachtet, welcher mit der Zeit immer deutlicher wuchs, und zwar zeigte sich die belichtete Lösung heller, d.h. ärmer an freiem Jod, wie später durch Titrierung erwiesen, als die unbelichtete. Hieraus kann man schliessen, dass das Licht auf die Jodabscheidungsreaktion hemmend einwirkt. Der Wärmeeffekt des auffallenden Lichtes kann hier ausser Betracht gelassen werden, da es darauf befördernd wirken muss.

Die Lösungen von Ferroammoniumsulfat und von Jod-Jodkalium wurden auf dieselbe Weise behandelt. Bald wurde die belichtete Lösung stark entfärbt, während die unbelichtete fast unverändert blieb, obwohl die Temperatur der letzteren, in warmes Wasser eingetaucht, stets ein wenig höher als diejenige der ersteren gehalten war. Also übt das Licht auf diese Reaktion einen beschleunigenden Einfluss aus.

Es wurde belichtet das Gemisch von Eisenalaun- und Jodkaliumlösung, welches im Dunkeln den Gleichgewichtszustand erreicht hatte. Bald wurde die Farbe der Lösung viel heller, wie es schon die oben erhaltenen Ergebnisse andeuten. Man kann auch hier den Wärmeeffekt des Lichtes versäumen, da seine Wirkung eine entgegengesetzte sein muss. Im Dunkeln kehrte diese heller gewordene Lösung wiederum zum Anfangszustande zurück. Daher folgt, dass die Nuance, d.h. die freie Jodmenge der Lösung durch Licht umkehrbar verändert wird. Ähnliche qualitative Versuche wurden mit anderen Ferrisalzen—Chlorid und Nitrat—und mit anderen Jodiden—Baryum- Zink- und Magnesium-

jodid—ausgeführt, immer mit derselben Lichtwirkung. Also muss diese Lichtwirkung eine allgemeine Eigenschaft der Ferrisalze und Jodide sein.

Quantitative Versuche.

Versuchsanordnung:—Als Lichtquelle diente eine 100 Volt—1500 Watt elektrische Glühlampe mit Stickstofffüllung. Um die mit der Richtung veränderlichen Lichtintensitäten auszugleichen, wurde die Lampe um ihre Symmetrieachse langsam gedreht. In einem Thermostat, der auf 25° gehalten war, wurden die Probierröhren, die die zu belichtende Lösung enthielten, am Stativ festgestellt, zu denen das Licht durch ein mit Glasplatte versehenes seitliches Fenster gelangte.

Die zum Versuche verwendeten Lösungen sind die folgenden:

1. Eisenalaun 0·04 molar; Ammoniumsulfat 0·02 molar; Schwefelsäure 0·1 normal;
2. Jodkalium 0·09 molar;
3. Ferroammoniumsulfat 0·04 molar; Schwefelsäure 0·1 normal;
4. Jod 0·04 normal; Jodkalium 0·05 molar; Kaliumsulfat 0·02 molar.

Zum Versuche wurde stets die aus gleichen Volumina 1. und 2. resp. 3. und 4. Lösung hergestellte gemischte Lösung gebraucht. Da ihre Zusammensetzung je einer Seite der Gleichung

$$2FeNH_4(SO_4)_2 + (NH_4)_2SO_4 + 2KJ + 2.5KJ + 5H_2SO_4$$
$$= 2Fe(NH_4)_2(SO_4)_2 + K_2SO_4 + J_2 + 2.5KJ + 5H_2SO_4$$

entspricht, so sollen die Gemische zu demselben Dunkelgleichgewicht führen. Der Einfachheit halber soll hernach die erstere gemischte Lösung als $Fe''' + J'$—Lösung und die letztere als $Fe'' + J$—Lösung bezeichnet werden.

Analysenmethode.—Gelegentlich wurde 5 ccm reagierende Lösung mit Pipette herausgenommen, mit 100 ccm Wasser verdünnt und mit ca. 1/200 n Natriumthiosulfatlösung titriert. Da Jod durch Einwirkung vom Ferrisalze auf Jodid beständig frei gemacht wird, so kann die Titrierung nicht sehr genau sein.

Titrimetrische Verfolgung des Reaktionsverlaufes bestätigte die qualitativen Versuchsergebnisse, und dazu wurde beobachtet, dass sowohl in $Fe''' + J'$— wie auch in $Fe'' + J$—Lösung sich ein Lichtgleichgewicht einstellte, welches an freiem Jod ärmer als das Dunkelgleichgewicht war.

Es wurde untersucht, ob das Lichtgleichgewicht von beiden Seiten her erreicht werden kann. Versuche wurden mit drei verschieden verdünnten $Fe''' + J' -$Lösungen:

I. $Fe''' + J' -$Lösung,
II. 1·6 fach verdünnte Lösung I,
III. 2 fach verdünnte Lösung I,

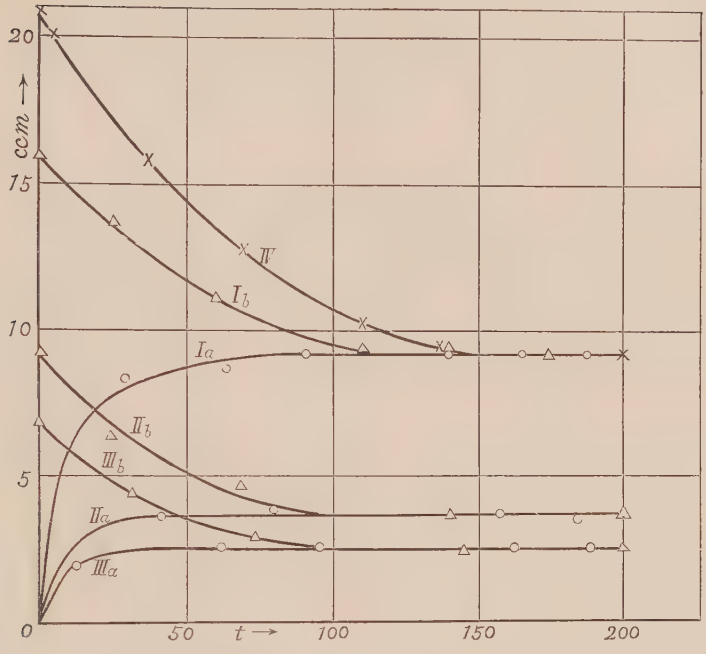
und mit $Fe'' + J -$Lösung (IV) ausgeführt. Die ersten drei Lösungen wurden im Dunkeln über Nacht gelassen und dann mit den frisch hergestellten vier Lösungen dem Licht ausgesetzt. Die zeitliche Veränderung des freien Jodes wurde jeweilig durch Titrierung messend verfolgt. Die folgende Tabelle I und Fig. 1 veranschaulichen den Verlauf der Reaktionen. In jeder Spalte unter t steht die Zeit in Minuten und unter ccm. die zur Titrierung verbrauchte Thiosulfatlösung in ccm.

Tabelle I.

I				II			
a frisch gemischt		b Vorabend gemischt		a frisch gemischt		b Vorabend gemischt	
t	ccm	t	ccm	t	ccm	t	ccm
0	0	0	16·0	0	0	0	9·25
29	8·27	25	13·7	41	3·65	24	6·37
63	8·69	60	11·1	79	3·85	68	4·65
90	9·20	110	9·4	157	3·69	140	3·68
140	9·20	140	9·4	184	3·60	200	3·65
165	9·22	174	9·2	200	3·70		

III				IV
a frisch gemischt		b Vorabend gemischt		frisch gemischt
t	ccm	t	ccm	t	ccm
0	0	0	6·87	0	20.8
12	2·00	31	4·40	37	15·8
61	2·57	73	2·95	69	12·8
94	2·60	145	2·45	110	10·3
162	2·47	200	2·46	137	9·5
188	2·60			200	9·2

Fig. 1.



Aus diesen Ergebnissen sieht man gleich, dass das Lichtgleichgewicht von beiden Seiten her erreicht werden kann.

Um die Abhängigkeit des Lichtgleichgewichts von der Lichtintensität erkennen zu können, wurde ein Versuch mit der $Fe''' + J' -$ Lösung ausgeführt. Die Lichtabschwächung erzielte man durch Einschaltung von einem oder mehreren Papierstücken zwischen die Probierröhren und die Lampe. Das Ergebnis ist in die Tabelle II gefasst, wo die Bezeichnung [J], welche auch in den folgenden Tabellen gebraucht wird, das in 1 Liter der Lösung enthaltene freie Jod in mgr-Äquivalenten gibt.

Tabelle II.

Zahl der Papierstücke	[J]
0	8·85
1	10·0
2	10·4

Beiläufig sei hier die unter der hellen Sonne sich einstellenden Lichtgleichgewichte neben den im Lampenlichte und im Dunkeln erreichten hinzugefügt. Die Versuche wurden mit den vorigen drei $Fe^{\cdots} + J' -$Lösungen ausgeführt. In der letzten Spalte der Tabelle III befindet sich totales Jod pro Liter in mgr-Äquivalenten.

Tabelle III.

Lösungen	Lichtgleichgewichte im Sonnenlicht erreichte [J]	Lichtgleichgewichte im Lampenlicht erreichte [J]	Dunkelgleichgewichte [J]	Totales Jod
I	4·45	8·85	15·4	20·0
II	1·38	3·72	9·4	12·5
III	0·92	2·55	7·1	10·0

Somit ist es klar, dass das Lichtgleichgewicht von der Lichtintensität abhängt.

Dass dies Lichtgleichgewicht ein wahres ist, kann man in der folgenden Weise beweisen, da in solchem Falle der Dunkelzustand durch intermittierende Belichtung und Verdunkelung in einem hermetisch verschlossenen Raume zu wiederholten Malen sich wiederherstellen lassen muss.[1] Die durch Kochen von Luft befreite Eisenalaun- und Jodkaliumlösung wurde im CO_2-Atmosphäre gemischt und in die Probierröhren eingeschlossen, deren Luft vorher durch CO_2-Strom gedrängt war. Die Befreiung der Lösung von Luft vor der Belichtung ist darum notwendig, weil andernfalls Sauerstoff unter Mitwirkung des Sonnenlichts aus Jodkalium Jod frei machen kann. Das Lampenlicht ist aber viel weniger wirksam und in den allen folgenden Versuchen soll seine Wirkung vernachlässigt werden. Die Lösungen wurden verschiedene Tage der hellen Sonne ausgesetzt und dann verdunkelt im Thermostat eingetaucht. Nach Einstellung des Dunkelgleichgewichts wurden ihre Jodmengen titriert. Die Resultate stimmen gut überein, wie die Tabelle IV zeigt.

[1] Luther u. Plotnikow, Zts. f. phys. Chem. **61**, 513, (1907).

Tabelle IV.

Belichtungsdauer in Tagen	[J]
1	17·66
8	17·64
11	17·80

Falls das Licht irgend einen besonderen von der Dunkelreaktion verschiedenen Vorgang zustande bringt, so muss der Reaktionsverlauf des belichteten Systems von dem Moment der Verdunkelung an ein anderer sein als der des unbelichteten, welches bei jenem Momente dieselbe Menge des freien Jodes besitzt. Insofern dieser Missklang des Reaktionsverlaufs nicht vorkommt, kann man wohl sicher annehmen, dass die vermutete spezielle Lichtreaktion nicht in Frage kommt, und dass die betreffende photochemische Reaktion eine einfache ist. Allerdings ist hierbei nicht ausgeschlossen, dass eine gewisse Veränderung bei Belichtung augenblicklich im reagierenden Systeme eintritt und bei der Verdunkelung auch augenblicklich verschwindet. Was hier uns interessiert, das sind Reaktionen mit messbarer, endlicher Geschwindigkeit.

Die nachstehenden Tabellen V und VI geben den Verlauf der Dunkelreaktionen ohne und nach Vorbelichtung an, und zwar bezieht sich die erstere auf die $Fe^{\cdots}+J'-$Lösung und die letztere auf die $Fe^{\cdot\cdot}+J-$Lösung. Die beistehende Fig. 2 veranschaulicht diese Reaktionsverlaufe.

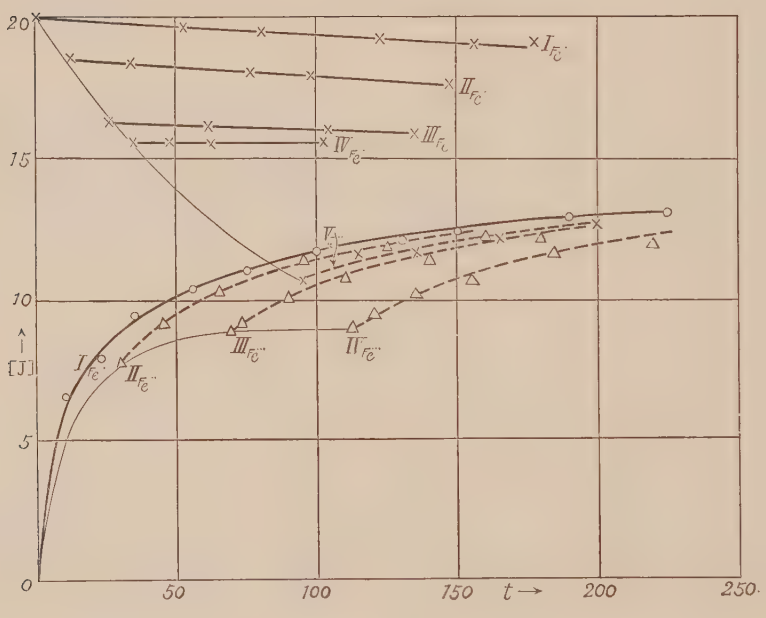
Tabelle V.

I		II		III		IV	
t	[J]	t	[J]	t	[J]	t	[J]
stets	dunkel	0	belichtet	0	belichtet	0	belichtet
0	0	0	0	0	0	0	0
22	7·9	30	verdunkelt	66	verdunkelt	111	verdunkelt
35	9·3	30	7·7	66	8·7	111	8·9
55	10·4	45	9·0	73	9·1	120	9·4
75	11·0	65	10·1	90	10·0	135	10·1
100	11·6	95	11·4	110	10·7	155	10·6
130	12·0	125	11·8	140	11·3	185	11·6
150	12·3	160	12·2	180	12·1	220	11·8

Tabelle VI.

I		II		III		IV		V	
t	[J]	t	[J]	t	[J]	t	[J]	t	[J]
stets	dunkel	0	belichtet	0	belichtet	0	belichtet	0	belichtet
0	20·0	0	20.0	0	20.0	0	20.0	0	20.0
52	19·6	12	verdunkelt	26	verdunkelt	33	verdunkelt	95	verdunkelt
80	19·5	12	18·5	26	16·3	33	15·6	95	10·7
122	19·2	33	18·4	61	16·1	47	15·6	115	11·6
156	19·0	76	18·0	104	16·0	62	15·5	135	11·6
178	19·1	98	17·9	135	15·8	102	15·5	165	12·1
		147	17·6					200	12·6

Fig. 2.



Wie aus den angeführten Tabellen und Kurven ersichtlich, verlaufen die Dunkelreaktionen nach Vorbelichtung fast identisch wie die

eigentlichen Dunkelreaktionen. Diese Übereinstimmung ist bei der $Fe^{\cdots}+J-$Dunkelreaktion wegen ihrer kleinen Reaktionsgeschwindigkeit ohne weiteres klar; bei der $Fe^{\cdots}+J'-$Dunkelreaktion ist sie dadurch veranschaulicht, dass die punktierten Kurven, welche durch die der Zeitachse parallele Verschiebung mit den entsprechenden Teilen der $Fe^{\cdots}+J'-$Dunkelreaktionskurve, (Kurve $I_{Fe^{\cdots}}$, Fig. 2), zusammenfallen sollen, die beobachteten Punkte gut verbinden. Bei einer verlängerteren Vorbelichtung wird die Dunkelreaktion etwas langsamer verlaufen. Bei genügend starker Säurekonzentration ist aber diese Unstimmigkeit nicht zu beobachten. Den Grund dieser Erscheinung hat man wohl in der Hydrolyse der Ferrisalze zu suchen.

Es sei hier noch betont, dass $Fe^{\cdot\cdot}+J-$Lösung, die im Lichte den dem Dunkelgleichgewicht entsprechenden Jodgehalt besitzt, selbst bei Verdunkelung ihren Zustand nicht verändert, (Kurve $IV_{Fe^{\cdot\cdot}}$, Fig. 2).

Es wurde eine weitere Prüfung der Einfachheit der Lichtreaktion dadurch erhalten, indem die Reaktionsverlaufe der von Anfang an belichteten und der von einem Momente der Dunkelreaktion an belichteten Systeme mit einander verglichen wurden. Die folgenden Tabellen VII und VIII enthalten die Versuchsergebnisse, und zwar bezieht sich die erstere auf die $Fe^{\cdots}+J'-$Lösung und die letztere auf die $Fe^{\cdot\cdot}+J-$Lösung. Die angeführte Fig. 3 macht diese Ergebnisse anschaulich. Die punktierten Kurven haben auch hier dieselbe Bedeutung wie oben. Nur die Normalkurve ist die der $Fe^{\cdot\cdot}+J-$Lichtreaktion (Kurve $I_{Fe^{\cdot\cdot}}$, Fig. 3).

Tabelle VII.

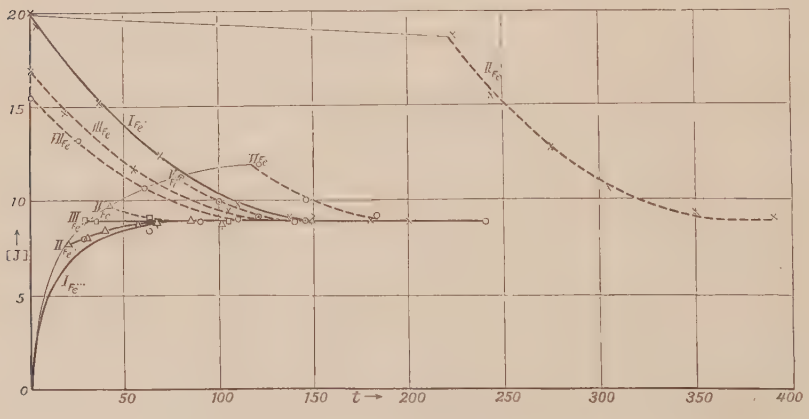
I		II		III		IV	
t	[J]	t	[J]	t	[J]	t	[J]
stets	bleichtet	0	dunkel	0	dunkel	0	dunkel
0	0	0	0	0	0	0	0
29	8·00	20	belichtet	29	belichtet	42	belichtet
63	8·40	20	7·71	29	9·00	42	9·70
90	8·84	30	8·07	35	8·92	67	8·88
140	8·84	40	8·41	63	9·09	101	8·67
165	8·84	57	8·70	105	8·92	149	8·89
187	8·75	85	8·89			200	8·84

V		VI		VII	
t	[J]	t	[J]	t	[J]
0	dunkel	0	dunkel	16·5 Stunden im Dunkeln gelassen	
0	0	0	0	0	belichtet
80	belichtet	121	belichtet	0	15·4
80	11·2	121	11·9	25	13·2
100	9·90	146	10·0	60	10·6
121	9·12	183	9·18	110	9·03
145	8·85	240	8·78	140	9·08
170	8·87	288	8·90	174	8·84

Tabelle VIII.

I		II		III	
t	[J]	t	[J]	t	[J]
stets	belichtet	0	dunkel	16.5 Stunden im Dunkeln gelassen	
0	20·0	0	20·0	0	belichtet
3·5	19·3	224	belichtet	0	16·9
37	15.2	224	18·7	18	14·7
69	12·3	244	15·5	55	11·6
110	9·82	275	12·8	105	9·42
137	9·12	305	10·6	150	9·13
200	8·83	350	9·34	180	8·94
		390	9·00		

Fig. 3.



So stimmen die beiden Reaktionsverlaufe vollkommen überein. Hier muss auch hervorgehoben werden, dass die $Fe''' + J'$—Lösung, die im Dunkeln den dem Lichtgleichgewichte entsprechenden Jodgehalt besitzt, ihren Zustand auch bei Belichtung unverändert beibehält (Kurve $III_{Fe'''}$, Fig. 3).

Es wurde weiter versucht, die bisher durch die analytische Methode ausgeführten Versuche auch durch andere physiko-chemische Methoden zu wiederholen, um die Einfachheit der betreffenden photochemischen Reaktion nochmals festzustellen. Aber die Veränderung der Viskozität, der Oberflächenspannung, sowie auch des elektrischen Leitvermögens war zu gering, um sie erfolgreich messend zu verfolgen. Nur die Veränderung des Farbentons war die einzige, wie schon oben angedeutet wurde. Auch durch quantitative kolorimetrische Versuche wurden ganz ähnliche Ergebnisse erhalten, welche aber einfachheitshalber hier nicht wiedergegeben werden.

Wie bereits gesagt, vermindert das Licht den Jodgehalt der im Dunkelgleichgewicht befindlichen Lösung. Was soll dann diese Verminderung bedeuten? Man darf nicht ohne jedes Bedenken schliessen, dass das Gleichgewicht

$$Fe''' + J' \rightleftarrows Fe'' + J$$

durch Licht bloss nach der linken Seite hin verschoben wird. Denn wir sind nicht imstande, die Möglichkeit auszuschliessen, dass das titrierbare Jod durch irgend eine besondere Lichtwirkung, ohne Ferrosalze zu oxydieren, reversibel in einen untitrierbaren Zustand übergegangen sei. Einige Gegenbeweise dafür haben wir schon durch die kinetische Verfolgung der Reaktion erhalten. Es kann aber eine unmittelbarere Methode sein, die Abnahme des freien Jodes mit der des Ferrosalzes analytisch zu identifizieren. Zu diesem Zwecke muss man die Mengen des freien Jodes und des Ferrosalzes in einer und derselben Lösung analytisch bestimmen. Bei der Titration des Jodes mit Thiosulfat entstehen Tetrathionat und Jodid, welche ihrerseits durch Permanganat oxydiert werden. Um die nachherige Titrierung des Ferrosalzes mit Permanganat ausführen zu können, müssen diese schädlichen Salze beseitigt werden. Zur Beseitigung des Tetrathionats gibt es kein bequemes Mittel. Wenn man aber Sulfit anstatt des Thiosulfats anwendet, so kann man diesen Übelstand vermeiden. Betreffs des Jodids kann man es durch Fällung mit Silbernitrat abschaffen. Reduktion des Silbernitrats durch Ferrosalz kann durch Vermeidung des Über-

schusses des ersteren und durch geringe Konzentration des letzteren Salzes unterdrückt werden. Silberjodid wurde abgefiltriert, um das scharfe Titrierungsende zu erzielen. Diese Analysenmethode lieferte zum vorliegenden Zwecke hinreichend genaue Resultate.

Die $Fe''' + J' -$Lösung, die im Dunkeln zum Gleichgewicht gelangt war, wurde dem Lichte ausgesetzt. Wie die nachstehende Tabelle IX veranschaulicht, wird die durch Licht hervorgerufene Abnahme des freien Jodes stets von der gleichzeitigen entsprechenden Abnahme des Ferrosalzes begleitet, und bei Verdunkelung nehmen diese beiden Stoffe auch in äquivalenten Mengen zu.

Tabelle IX.

t	[J]	[Fe'']
0	belichtet	
0	15·5	15·5
38	—	12·9
51	12·4	12·3
59	12·3	12·2
75	11·7	11·8
136	10·3	10·4
141	10·6	10·6
152	10·3	10·6
170	verdunkelt	
175	10·8	11·4
187	11·6	11·7
195	12·0	11·9
204	11·9	12·0
237	13·6	13·4

Die bisher erhaltenen Versuchsergebnisse gestatten uns nun, die folgende Schlussfolgerung daraus zu ziehen: Die durch Licht herbeigeführte umkehrbare Verschiebung des Dunkelgleichgewichts muss einfach im Sinne der Gleichung

$$Fe''' + J' \underset{\text{Licht}}{\overset{\text{Dunkel}}{\rightleftarrows}} Fe'' + J$$

geschehen. Ob sich hier J_3' oder J_2 beteiligt, ist eine Frage, die durch weitere Untersuchung zu entscheiden ist.

Jod zeigt unter dem Einfluss des Lichtes gewöhnlich die Neigung von dem gebundenen Zustande sich zu entbinden, wie bei der photo-

chemischen Zersetzung des Jodsilbers und des Jodwasserstoffgases. Nach Coehn[1] wird das Jodwasserstoffgas sowohl durch sichtbares als auch durch langwelliges ultraviolettes Licht vollständig zersetzt, während es durch kurzwelligeres (254—220 $\mu\mu$) Licht aus seinen Komponentenelementen, wenn auch sehr gering, gebildet wird. Es ist somit eine merkwürdige Tatsache, dass im vorliegenden Falle langwelliges sichtbares Licht auf Jodidbildung günstig einwirkt. Es soll noch betont werden, dass bei gewöhnlichen Temperaturen sowohl im Dunkeln als auch im Lichte der Gleichgewichtszustand der betreffenden Reaktion aus messbaren Mengen von jeden einzelnen Reaktionsteilnehmer besteht, eine Bedingung, die nur selten erfüllt wird, wie bei der allotropen Umwandlung des Schwefels[2] und bei der Bromaddition an α-Phenylzimmtsäureniril.[3]

Zusammenfassung.

Es wurde festgestellt,

1. dass die Reaktion zwischen Ferrisalzen und Jodiden, resp. zwischen Ferrosalzen und freiem Jod gegen Licht—der Sonne und der elektrischen Glühlampe—eine deutliche Empfindlichkeit zeigt, und zwar

2. dass die erstere dadurch verzögert und die letztere befördert wird. Ferner

3. dass das Dunkelgleichgewicht durch Licht, dem gewöhnlichen Verhalten der Jodverbindung und auch dem Effekt der Temperaturerhöhung entgegen, nach Jodidbildung hin verschoben wird, und

4. dass sich ein wohl definiertes von der Lichtintensität abhängiges Lichtgleichgewicht einstellt. Und schliesslich

5. dass die betreffende photochemische Reaktion von äusserst einfacher Natur ist.

Die weitere Untersuchung der vorliegenden Reaktion ist nun noch im Gang.

Es möge hier bemerkt werden, dass zur Ausführung dieser Arbeit die Benutzung des Regierungszuschusses zur Beförderung der Naturwissenschaften gestattet wurde.

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1 Coehn u. Stuchardt, Zts. f. phys. Chem. **91**, 722, (1916).

2 Wigand, ebenda, **78**, 208; **77**, 423, (1911).

3 Plotnikow, Zts. f. wiss. Photo. **19**, 1, (1919).

On the Congruence Conjugate to a Surface.

By

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(Received February 10, 1922.)

Introduction.

Darboux has proved the following theorem.[1]

When, passing through all the straight lines in a fixed plane, tangent planes to two surfaces (S), (S_1) are drawn, the straight lines joining the contact points form a congruence whose developables cut both surfaces (S) and (S_1) in a conjugate net.

Let us now suppose that a surface (S) with a conjugate net on it is given and consider the surface (S_1) which is in the following relation to (S).

1° There is a congruence whose developables cut (S) in given conjugate net and (S_1) also in a conjugate net.

2° Intersections of the tangent planes of (S) and (S_1) at the corresponding points describe a plane.

We shall say that such surfaces as (S_1) are in the relation (D) to (S).

We shall determine all the surfaces which are in the relation (D) to (S) and investigate their geometrical properties.

We assume the given surfaces is not developable surface and take as the parameter-curves the given conjugate net on (S).

Green has proved that in this case the homogeneous coordinates (x) of the points on (S) are a fundamental system of the solutions of a

1 Darboux, "Theorie générale des surfaces" II. Chap. X.

completely integrable system of partial differential equations of the second order.[1]

$$\left.\begin{aligned} \frac{\partial^2 x}{\partial u^2} &= a\frac{\partial^2 x}{\partial v^2} + b\frac{\partial x}{\partial u} + c\frac{\partial x}{\partial v} + dx \\ \frac{\partial^2 x}{\partial u \partial v} &= b_1\frac{\partial x}{\partial u} + c_1\frac{\partial x}{\partial v} + d_1 x \end{aligned}\right\} \qquad \text{(I)}$$

where a, b, c, d, b_1, c_2 and d_1 are functions of u, v.

We shall call, hereafter, (I) the equations of the given surface.

The higher derivatives of (x) can be expressed by linear combinations of $\frac{\partial^2 x}{\partial v^2}$, $\frac{\partial x}{\partial u}$, $\frac{\partial x}{\partial v}$ and x by (I)

In particular, we have

$$\frac{\partial^3 x}{\partial u \partial v^2} = a_2\frac{\partial^2 x}{\partial v^2} + b_2\frac{\partial x}{\partial u} + c_2\frac{\partial x}{\partial v} + d_2 x$$

$$\frac{\partial^3 x}{\partial v^3} = a_3\frac{\partial^2 x}{\partial v^2} + b_3\frac{\partial x}{\partial v} + c_3\frac{\partial x}{\partial v} + d_3 x$$

where

$$a_2 = c_1, \quad b_2 = b_1^2 + \frac{\partial b_1}{\partial v}, \quad c_2 = b_1 c_1 + \frac{\partial c_1}{\partial v} + d_1$$

$$d_2 = b_1 d_1 + \frac{\partial d_1}{\partial v}, \quad a_3 = \frac{1}{a}\left(a b_1 - c - \frac{\partial a}{\partial v}\right), \quad b_3 = \frac{1}{a}\left(b_1 c_1 + \frac{\partial b_1}{\partial u} - \frac{\partial b}{\partial v} + d_1\right)$$

$$c_3 = \frac{1}{a}\left\{b_1 c + c_1(c_1 - b) + \frac{\partial c_1}{\partial u} - \frac{\partial c}{\partial v} - d\right\}$$

$$d_3 = \frac{1}{a}\left\{b_1 d + d_1(c_1 - b) + \frac{\partial d_1}{\partial u} - \frac{\partial d}{\partial v}\right)$$

The functions $a, b, c, d, b_1, c_1, d_1$, are restricted by conditions of the integrability one of which is

$$\frac{\partial}{\partial v}(b + 2c_1) = \frac{\partial}{\partial u}(a_3 + b_1).$$

Hence we can find a function p such that

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[1] Amer. J. Math., **37**, 215–246 (1915).

$$b+2c_1=\frac{\partial p}{\partial u},\quad a_1+b_1=\frac{\partial p}{\partial v}$$

As (x) are the fundamental system of solutions of (I), the general solution of (I) is

$$x=C_1x_1+C_2x_2+C_3x_3+C_4x_4$$

where C_1, C_2, C_3, C_4, are arbitarily constants.

Therefore we know that all the surfaces whose equation are (I), are the projective transforms of (S).

Denote by B and C the functions

$$B=b_1-\frac{1}{4}\frac{\partial p}{\partial v}-\frac{3}{8a}\frac{\partial a}{\partial v}$$

$$C=c_1-\frac{1}{4}\frac{\partial p}{\partial v}+\frac{1}{8a}\frac{\rho a}{\partial u}$$

Then the functions a, B and C are invariant of the transformation

$$\bar{x}=\lambda(u_1v)x.$$

which do not alter, evidently, the surface nor the parameter-curves.

§ I. A congruence Γ whose developables correspond to given conjugate net.

Consider two points (ρ), (σ) such that

$$(\rho)=\left(\frac{\partial x}{\partial u}\right)+\mu(x)$$

$$(\sigma)=\left(\frac{\partial x}{\partial v}\right)+\lambda(x)$$

The points (ρ), (σ) evidently lie on the tangent lines at (x) of the curves C_u ($v=$const.), and C_v ($u=$const.) respectively which meet at (x).

When (x) moves on the given surface, the lines (ρ) (σ) form a congruence which is called by Green conguence Γ.

If the lines (ρ) (σ) form a developable surface when (x) moves along a curve C_u, the four point $\left(\frac{\partial\rho}{\partial u}\right)$, $\left(\frac{\partial\sigma}{\partial u}\right)$, (ρ) and (σ) must lie in a plane.

But we have

$$\left(\frac{\partial \rho}{\partial u}\right)=\left(\frac{\partial^2 x}{\partial u^2}\right)+u\left(\frac{\partial x}{\partial u}\right)+\frac{\partial \mu}{\partial u}(x),$$

$$\left(\frac{\partial \sigma}{\partial u}\right)= \left(b_1+\lambda\right)\left(\frac{\partial x}{\partial u}\right)+c_1\left(\frac{\partial x}{\partial v}\right)+\left(d_1+\frac{\partial \lambda}{\partial v}\right)(x).$$

Hence the points $\left(\frac{\partial \sigma}{\partial u}\right)$, (ρ), (σ) are in the tangent plane at (x), while the point $\left(\frac{\partial \rho}{\partial u}\right)$ is not on the tangent plane, for the point $\left(\frac{\partial^2 x}{\partial u^2}\right)$ does not lie in the tangent plane unless the surface (S) is developable.

Therefore if the points $\left(\frac{\partial \rho}{\partial u}\right)$, $\left(\frac{\partial \sigma}{\partial u}\right)$, (ρ), (σ) lie in a plane, the three points $\left(\frac{\partial \sigma}{\partial u}\right)$, (ρ), (σ) must lie in a straight line, that is, $\left(\frac{\partial \sigma}{\partial u}\right)$ must be of the form

$$\omega_1(\rho)+\omega_2(\sigma)$$

Eliminating ω_1 and ω_2 we have

$$\frac{\partial \lambda}{\partial u}-\lambda\mu-c_1\lambda-b_1\mu+d_1=o$$

Similarly if the lines (ρ) (σ) describe a developable surface when (x) moves along a curve C_v, we must have

$$\frac{\partial \mu}{\partial v}-\lambda\mu-c_1\lambda-b_1\mu+d_1=o$$

Hence we know that the developables of the congruence formed by lines (ρ) (σ) correspond to the given conjugate net, if, and only if

$$\frac{\partial \lambda}{\partial u}=\frac{\partial u}{\partial v}$$

$$\frac{\partial^2 \theta}{\partial u \partial v}=b_1\frac{\partial \theta}{\partial u}+c_1\frac{\partial \theta}{\partial v}+d_1\theta \quad \text{.....(II)}$$

where θ is a function such that

$$\lambda=-\frac{1}{\theta}\frac{\partial \theta}{\partial v}, \qquad \mu=-\frac{1}{\theta}\frac{\partial \theta}{\partial u}.$$

Hence the points (ρ), (σ) which satisfy the above conditions may be expressed in the form :

$$\left.\begin{aligned}(\rho)&=\theta\left(\frac{\partial x}{\partial u}\right)-\frac{\partial\theta}{\partial u}(x)\\(\sigma)&=\theta\left(\frac{\partial x}{\partial v}\right)-\frac{\partial\theta}{\partial v}(x)\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where θ is a solution of the partial differential equation of the second order (II).

In this case the points (ρ), (σ) are focal points, for we have by differentiation

$$\left(\frac{\partial\rho}{\partial v}\right)=\left(b_1+\frac{1}{\theta}\frac{\partial\theta}{\partial v}\right)(\rho)+\left(c_1-\frac{1}{\theta}\frac{\partial\theta}{\partial u}\right)(\sigma)$$

$$\left(\frac{\partial\sigma}{\partial u}\right)=\left(b_1-\frac{1}{\theta}\frac{\partial\theta}{\partial v}\right)(\rho)+\left(c_1+\frac{1}{\theta}\frac{\partial\theta}{\partial u}\right)(\sigma)$$

We shall nextly find the condition that the line (ρ) (σ) given by (1) may describe a plane when (x) moves on the given surface (S).

If the lines (ρ) (σ) describe a plane the four points (σ), (ρ), $\left(\frac{\partial\sigma}{\partial v}\right)$, $\left(\frac{\partial\rho}{\partial u}\right)$ must necessarily lie in a plane.

But we have

$$\left(\frac{\partial\sigma}{\partial v}\right)=\theta\left(\frac{\partial^2 x}{\partial v^2}\right)-\frac{\partial^2\theta}{\partial v^2}(x)$$

$$\left(\frac{\partial\rho}{\partial u}\right)=a\theta\left(\frac{\partial^2 x}{\partial v^2}\right)+b\theta\left(\frac{\partial x}{\partial u}\right)+c\theta\frac{\partial x}{\partial v}+\left(d\theta-\frac{\partial^2\theta}{\partial u^2}\right)(x)$$

Therefore if the points (σ), (ρ), $\left(\frac{\partial\sigma}{\partial v}\right)$, $\left(\frac{\partial\rho}{\partial u}\right)$ lie in a plane we must have

$$\begin{vmatrix} o & \theta & o & -\frac{\partial\theta}{\partial u}\\ o & o & \theta & -\frac{\partial\theta}{\partial v}\\ \theta & o & o & -\frac{\partial^2\theta}{\partial v^2}\\ a\theta & b\theta & c\theta & d\theta-\frac{\partial^2\theta}{\partial u^2}\end{vmatrix}=o,$$

or

$$\frac{\partial^2\theta}{\partial u}=a\frac{\partial^2\theta}{\partial v^2}+b\frac{\partial\theta}{\partial u}+c\frac{\partial\theta}{\partial v}+d\theta,$$

that is, θ must be a solution of the system of simultaneous partial differential equations (I).

The condition is sufficient

For θ being a solution of (I), is of the form

$$\theta=C_1x_1+C_2x_2+C_3x_3+C_4x_4.$$

Substituting this value of θ in (1) we know that both (ρ) and (σ) satisfy the equation

$$C_1z_1+C_2z_2+C_3z_3+C_4z_4=0$$

that is, the line (ρ) (σ) describes a plane

§ 2. Congruences conjugate to the surface.

As the point $\left(\frac{\partial^2x}{\partial v^2}\right)$ is out of the tangent plane at (x), any point can be expressen in the form

$$\omega_1\left(\frac{\partial^2x}{\partial v^2}\right)+\omega_2\left(\frac{\partial x}{\partial u}\right)+\omega_3\left(\frac{\partial x}{\partial v}\right)+\omega_4(x).$$

Therefore any line passing through the point (x) and is not on the tangent plane may be regarded as the line which passes through the point (x) and through the point (z) which is of the form

$$(z)=\left(\frac{\partial^2x}{\partial v^2}\right)+\lambda\left(\frac{\partial\lambda}{\partial u}\right)+\mu\left(\frac{\partial x}{\partial v}\right).$$

If λ and μ are given as the functions of u, v the lines (x) (z) form a congruence when the point (x) moves on the surface (S).

Now we shall determine λ and μ so that the developable of the congruence cut the surface in the curves C_u and C_v.

If the lines (x) (z) form a develpable when (x) moves along the curves C_u four points (x), (z), $\left(\frac{\partial x}{\partial u}\right)$, $\left(\frac{\partial z}{\partial u}\right)$ must lie in a plane, that is, $\left(\frac{\partial z}{\partial u}\right)$ must be of the form

$$\alpha(z)+\beta\left(\frac{\partial x}{\partial u}\right)+\gamma(x),$$

or of the form

$$\alpha\left(\frac{\partial^2 x}{\partial v^2}\right)+\beta'\left(\frac{\partial x}{\partial u}\right)+\alpha\mu\left(\frac{\partial x}{\partial v}\right)+\gamma(x).$$

But we have by differentiation

$$\left(\frac{\partial z}{\partial u}\right)=(a\lambda+a_2)\frac{\partial^2 x}{\partial v^2}+\left(b\lambda+b_1\mu+b_2+\frac{\partial\lambda}{\partial u}\right)\left(\frac{\partial x}{\partial u}\right)$$
$$+\left(c\lambda+c_1\mu+c_2+\frac{\partial\mu}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right)$$
$$+\left(d\lambda+d_1\mu+d_2\qquad\right)(x).$$

Therefore we must have

$$(a\lambda+a_2)\mu=c\lambda+c_1\mu+c_2+\frac{\partial\mu}{\partial u}$$

or

$$\frac{\partial\mu}{\partial u}-a\lambda u-\lambda c+c_2=o.$$

Similarly, if the lines (x) (z) form a developable surface when (x) moves along a curve C_v, we must have

$$\frac{\partial\lambda}{\partial v}-\lambda u+\lambda(b_1-a_3)+b_3=o.$$

Therefore, if the developables of the congruence formed by the lines (x) (z) cut the surface (S) in curves C_u and C_v, we must have

$$\frac{\partial\mu}{\partial u}-a\lambda u+\lambda c+c_2=o$$

$$\frac{\partial\lambda}{\partial v}-\lambda u+\lambda(b_1-a_3)+b_3=o.$$

Multiplying the second by $-a$ and adding, we have

$$\frac{\partial}{\partial u}(u-b_1)=\frac{\partial}{\partial v}(a\lambda-b-c_1).$$

But there is the relation

$$\frac{\partial}{\partial v}(b+2c_1)=\frac{\partial}{\partial u}(a_3+b_1)$$

Hence we have

$$\frac{\partial}{\partial u}(u+a_3)=\frac{\partial}{\partial v}(a\lambda+c_1).$$

This shows that there exists a function θ of u, v such that

$$a\lambda+c_1=-\frac{1}{\theta}\frac{\partial\theta}{\partial u}$$

$$u+a_3=-\frac{1}{\theta}\frac{\partial\theta}{\partial v}.$$

Substituting these values of λ, u in any one of (2), we have

$$\frac{\partial^2\theta}{\partial u\partial v}=-\left(b_1-\frac{1}{a}\frac{\partial a}{\partial v}\right)\frac{\partial\theta}{\partial u}-c_1\frac{\partial\theta}{\partial v}$$
$$+\left(d_1+\frac{\partial b_1}{\partial u}+\frac{\partial c_1}{\partial v}-\frac{\partial^2 p}{\partial u\partial v}+\frac{c_1}{a}\frac{\partial a}{\partial v}\right)\theta. \quad \text{.....(III)}$$

Hence we know that any congruence whose developables cut the surfaces (S) in the system of the curves C_u and C_v is that which is formed by the lines which pass through the point (x) and through the point (z) such that

$$(z)=\theta\left(\frac{\partial^2 x}{\partial v^2}\right)-\frac{1}{a}\left(\frac{\partial\theta}{\partial u}+c_1\theta\right)\left(\frac{\partial x}{\partial u}\right)-\left(\frac{\partial\theta}{\partial v}+a_3\theta\right)\left(\frac{\partial x}{\partial v}\right)\text{.....(3)}$$

where θ is a solution of the partial differential equation of the second order (III).

§ 3. Focal surfaces of the congruences conjugate to the given surface.

By differentiation of (z) given by (3), we know in virtue of (III)

$$\left.\begin{aligned}\left(\frac{\partial z}{\partial u}\right)&=\xi\left(\frac{\partial x}{\partial u}\right)+\eta(x)\\ \left(\frac{\partial z}{\partial v}\right)&=\xi'\left(\frac{\partial x}{\partial v}\right)+\eta'(x).\end{aligned}\right\}\text{.....(4)}$$

where

$$\xi=-\frac{1}{a}\frac{\partial^2\theta}{\partial u^2}-\frac{1}{a}\left(b+c_1-\frac{1}{a}\frac{\partial a}{\partial u}\right)\frac{\partial\theta}{\partial u}-b_1\frac{\partial\theta}{\partial v}$$
$$+\left(b_2-\frac{bc_1}{a}-\frac{\partial}{\partial u}\left(\frac{c_1}{a}\right)-b_1a_3\right)\theta$$

$$\xi' = -\frac{\partial^2\theta}{\partial v^2} - \frac{c_1}{a}\frac{\partial\theta}{\partial u} - a_3\frac{\partial\theta}{\partial v} + \left(c_3 - \frac{c_1^2}{a} - \frac{\partial a_3}{\partial v}\right)\theta$$

$$\eta = -\frac{d}{a}\frac{\partial\theta}{\partial u} - d_1 + \frac{\partial\theta}{\partial v} + \left(d_2 - \frac{c_1 d}{a} - d_1 a_3\right)\theta$$

$$\eta' = -\frac{d_1}{a}\frac{\partial\theta}{\partial u} + \left(d_3 - \frac{d_1 c_1}{a}\right)\theta$$

Eliminating (z) from (4), we have

$$\left.\begin{aligned} -\frac{\partial\xi}{\partial v} + \eta' &= b_1(\xi - \xi') \\ +\frac{\partial\xi}{\partial u} - \eta &= c_1(\xi - \xi') \\ \frac{\partial\eta}{\partial v} - \frac{\partial\eta'}{\partial u} &= d_1(\xi - \xi') \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

If we put

$$\zeta = \xi - \xi'$$

We know from (5)

$$\frac{\partial^2\zeta}{\partial u\partial v} = -b_1\frac{\partial\zeta}{\partial u} - c_1\frac{\partial\zeta}{\partial v} + \left(d_1 - \frac{\partial b_1}{\partial u} - \frac{\partial c_1}{\partial v}\right)\zeta \ldots\ldots\ldots\ldots (6)$$

that is, ζ is a solution of the partial differential equation adjointed to (II).

Any point (ρ) on the line (x) (z) is given by the equation

$$(\rho) = (z) + \lambda(x).$$

From (4) we have

$$\left(\frac{\partial\rho}{\partial u}\right) = (\xi + \lambda)\left(\frac{\partial x}{\partial u}\right) + \left(\frac{\partial\lambda}{\partial u} + \eta\right)(x)$$

$$\left(\frac{\partial\rho}{\partial v}\right) = (\xi' + \lambda)\left(\frac{\partial x}{\partial v}\right) + \left(\frac{\partial\lambda}{\partial v} + \eta'\right)(x)$$

If the point (ρ) is a focal point on the line (x) (z), one of the $\left(\frac{\partial\rho}{\partial u}\right)$ and $\left(\frac{\partial\rho}{\partial v}\right)$ must be of the form

$$a(x)+\beta(z)$$

or

$$\lambda=\xi \quad \text{or} \quad \lambda=\xi'.$$

Hence two focal points on the line $(x)\,(z)$ are given by

$$(\omega)=(z)-\xi(x)$$
$$(\nu)=(z)-\xi'(x).$$

From (4) and (5) we have

$$\left.\begin{aligned}
\left(\frac{\partial\omega}{\partial u}\right)&=-\left(\frac{\partial\zeta}{\partial u}+c_1\zeta\right)(x)\\
\left(\frac{\partial\omega}{\partial v}\right)&=-\zeta\left(\frac{\partial x}{\partial v}\right)+b_1\zeta(x)\\
\left(\frac{\partial\nu}{\partial\omega}\right)&=\zeta\frac{\partial x}{\partial\omega}-c_1\zeta(x)\\
\left(\frac{\partial\nu}{\partial v}\right)&=\left(\frac{\partial\zeta}{\partial v}+b_1\zeta\right)(x).
\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

The function ζ is a function of θ and its derivation to the second order with respect to u and also to v and satisfies a partial differential equation of the second order (6). Therefore we know that the equation $\zeta=o$ is satisfied by four particular solutions of (II).[1]

If we take one of those solitions, we have

$$\zeta=o\,;\quad \xi=\xi',$$

that is,

$$\omega=\nu$$
$$\frac{\partial\omega}{\partial u}=\frac{\partial\omega}{\partial v}=\frac{\partial\nu}{\partial u}=\frac{\partial\nu}{\partial v}=o.$$

Hence ω is independent of u and v, that is, the lines of the congruences corresponding to these solutions pass through a fixed point.

Returning to the general case eliminate (x) from the last two equations of (7) then we know ν is a solution of the partial differential equation of the second order

---

[1] Darboux, loc cit. II, p. 174.

$$\frac{\partial^2\Theta}{\partial u\partial v}=\left(b_1+\frac{1}{\zeta}\frac{\partial\zeta}{\partial v}\right)\frac{\partial\Theta}{\partial u}$$
$$+\left\{c_1+\frac{1}{b_1\zeta+\frac{\partial\zeta}{\partial v}}\frac{\partial}{\partial u}\left(b_1\zeta+\frac{\partial\zeta}{\partial v}\right)\right\}\frac{\partial\Theta}{\partial v} \quad \text{.....(IV)}$$

Reciprocally, if we take any solution Θ of (IV), the function θ given by

$$\theta=\frac{1}{b_1\zeta+\frac{\partial\zeta}{\partial v}}\frac{\partial\Theta}{\partial v} \quad \text{.....(V)}$$

is a solution of (II).

For differentiating the equation

$$\frac{\partial\Theta}{\partial v}=\theta\left(b_1\zeta+\frac{\partial\zeta}{\partial v}\right)$$

by u, we have by (IV).

$$\frac{\partial\theta}{\partial u}=\frac{1}{\zeta}\frac{\partial\Theta}{\partial u}+c_1\theta \quad \text{.....(8)}$$

Nextly differntiating this equation by v, we have by (V), (6) and (8)

$$\frac{\partial^2\theta}{\partial u\partial v}=b_1\frac{\partial\theta}{\partial u}+c_1\frac{\partial\theta}{\partial v}+d_1\theta.$$

From (V) and (9) we know that there are between θ and Θ the relation

$$\left.\begin{aligned}\frac{\partial\Theta}{\partial u}&=\zeta\left(\frac{\partial\theta}{\partial u}-c_1\theta\right)\\ \frac{\partial\Theta}{\partial v}&=\theta\left(\frac{\partial\zeta}{\partial v}+b_1\zeta\right)\end{aligned}\right\} \quad \text{.....(VI)}$$

In the above discussion we have assumed

$$\frac{\partial\zeta}{\partial v}+b_1\zeta\neq o$$

But, if

$$\frac{\partial\zeta}{\partial v}+b\zeta=o$$

combining this with

$$o=\frac{\partial}{\partial u}\left(\frac{\partial\zeta}{\partial v}+b_1\zeta\right)=-c_1\frac{\partial\zeta}{\partial v}+\left(d_1-\frac{\partial c_1}{\partial v}\right)\zeta.$$

we have

$$d_1+b_1c_1-\frac{\partial c_1}{\partial v}=o$$

In this case Darboux-Laplace transform of (S) must be reduced to a curve or a developable.

§ 4. Surfaces which are in the relation (D) to the given surface.

Any point (ρ) on the line (x) (z), being also a point on the line (x) (ν), can be expressed in the form

$$(\rho)=(\nu)+\lambda(x)=(\nu)+\frac{\lambda}{b_1\zeta+\frac{\partial\zeta}{\partial v}}\left(\frac{\partial\nu}{\partial\zeta}\right)$$

But the surface described by (ν) is one of the focal surfaces of the congruence. When the developables of this congruence cut the surface which is described by (ρ) in a conjugate net, (ρ) must satisfy the partial differential equation of the second order of the form.

$$\frac{\partial^2\rho}{\partial u\partial v}=p\frac{\partial\rho}{\partial u}+q\frac{\partial\rho}{\partial v}+r\rho$$

From this follows that λ must satisfy the equation

$$\frac{\lambda}{b_1\zeta+\frac{\partial\zeta}{\partial v}}=-\frac{\theta}{\frac{\partial\theta}{\partial v}} \quad \text{or} \quad \lambda=-\theta\frac{b_1\zeta+\frac{\partial\zeta}{\partial v}}{\frac{\partial\theta}{\partial v}}$$

where θ is a solution of (IV).[1]

But we have proved that $\dfrac{\frac{\partial\theta}{\partial v}}{b_1\zeta+\frac{\partial\zeta}{\partial v}}$ is a solution of (II).

Hence we know that all the surfaces such that developables of the same congruence cut them in a conjugate net and the given surface in the given conjugate net are given by

$$(\rho)=(\nu)-\frac{\theta}{\theta}(x)$$

---

[1] Darboux, loc. cit. Vol. II. p. 222.

where Θ is a solution of (IV) and θ is a solution of (II) and Θ and θ are connected by (V).

But when $\zeta=o$, as noticed in § 3 $\nu=$const., that is, the surface described by (ρ) is reduced to a point.

In this case we take as the form of (ρ)

$$(\rho)=\lambda(\nu)+(x) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

from which we have

$$\frac{\partial^2\rho}{\partial u\partial v}=b_1\frac{\partial\rho}{\partial u}+c_1\frac{\partial\rho}{\partial v}+d_1\rho+\left(\frac{\partial^2\lambda}{\partial u\partial v}-b_1\frac{\partial\lambda}{\partial u}-c_1\frac{\partial\lambda}{\partial v}-d_1\lambda\right)\nu$$

When the surface described by (ρ) satisfies our condition, we must have

$$\frac{\partial^2\lambda}{\partial u\partial v}=b_1\frac{\partial\lambda}{\partial u}+c_1\frac{\partial\lambda}{\partial v}+d_1\lambda.$$

Therefore also in this case, dividing (10) by λ, (ρ) can be expressed in the form

$$(\rho)=(\nu)+\frac{(x)}{\theta}$$

where θ is a solution of (II).

By differentiation from (9), we have

$$\left.\begin{aligned}\left(\frac{\partial\rho}{\partial u}\right)&=(\zeta\theta-\Theta)\frac{\partial}{\partial u}\left\{\frac{(x)}{\theta}\right\}\\ \left(\frac{\partial\rho}{\partial v}\right)&=-\Theta\frac{\partial}{\partial v}\left\{\frac{(y)}{\theta}\right\}.\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

Those formulas are given by Darboux by another method.

The points $\left(\frac{\partial\rho}{\partial u}\right)$, $\left(\frac{\partial\rho}{\partial v}\right)$ are on the tangent plane at (ρ) to its surface.

The points $\frac{\partial}{\partial u}\left\{\frac{(x)}{\theta}\right\}$, $\frac{\partial}{\partial v}\left\{\frac{(x)}{\theta}\right\}$ are on the tangent plane at (x) to the given surface.

But the equation (11) shows that the points $\left(\frac{\partial\rho}{\partial u}\right)$, $\left(\frac{\partial\rho}{\partial v}\right)$ coincide respectively with the points $\frac{\partial}{\partial u}\left\{\frac{(x)}{\theta}\right\}$, $\frac{\partial}{\partial v}\left\{\frac{(x)}{\theta}\right\}$.

Hence the line joining the points $\frac{\partial}{\partial u}\left\{\begin{matrix}(x)\\ \theta\end{matrix}\right\}$, $\frac{\partial}{\partial v}\left\{\begin{matrix}(x)\\ \theta\end{matrix}\right\}$ is the intersection of the corresponding tangent planes to the given surface and to a surface given by (9).

But we have shown in § 1 that when the line joining the points $\frac{\partial}{\partial u}\left\{\begin{matrix}(x)\\ \theta\end{matrix}\right\}$, $\frac{\partial}{\partial v}\left\{\begin{matrix}(x)\\ \theta\end{matrix}\right\}$ describes a plane as the point (x) moves on the given surfaces (S), θ must be a solution of the simultaneous partial differential equations (I).

Hence we know that surfaces which are in the relation (D) to the given surface are given by

$$(\rho)=\nu-\frac{\Theta}{\theta}(x).$$

where θ is a solution of the simultaneous partial differential equation (I).

Put

$$x=\theta y \qquad \text{.....(12)}$$

Then y satisfies also a system of the simultaneous partial differential equations of the form (I). These equations, being satisfied by $y=1$ (which corresponds to the solution $x=\theta$ of the original equations), must be of the form

$$\frac{\partial^2 y}{\partial u^2}=a\frac{\partial^2 y}{\partial v^2}+\beta\frac{\partial y}{\partial u}+\gamma\frac{\partial y}{\partial v}$$

$$\frac{\partial^2 y}{\partial u\partial v}=\beta_1\frac{\partial y}{\partial u}+c_1\frac{\partial y}{\partial v}.$$

As we have noticed in the introduction, the surface and also the parameter-curves are not altered by this transformation (12) and also the functions a, B, and C.

From (12) and (11) we have

$$\left(\frac{\partial\rho}{\partial u}\right)=\left(\zeta\theta-\theta\right)\left(\frac{\partial y}{\partial u}\right)$$

$$\left(\frac{\partial\rho}{\partial v}\right)=-\theta\left(\frac{\partial y}{\partial v}\right).$$

By differentiation of the above equation, we know that (ρ) satisfies the following system of the simultaneous partial differential equations of the second order which are completely integrable.

$$\frac{\partial^2\rho}{\partial u^2}=a\left(\frac{\Theta-\zeta\theta}{\Theta}\right)\frac{\partial^2\rho}{\partial v^2}+\left(\beta+\frac{\frac{\partial}{\partial u}(\Theta-\zeta\theta)}{\Theta-\zeta\Theta}\right)\frac{\partial\rho}{\partial u}$$

$$+\frac{\Theta-\zeta\theta}{\Theta}\left(\gamma-\frac{a}{\Theta}\frac{\partial\Theta}{\partial v}\right)\frac{\partial\rho}{\partial v}\Big\}\cdots\cdots\cdots\cdots(\text{VII})$$

$$\frac{\partial^2\rho}{\partial u\partial v}=\left\{\beta_1+\frac{\frac{\partial}{\partial v}(\Theta-\zeta\theta)}{\Theta-\zeta\theta}\right\}\frac{\partial\rho}{\partial u}+\left(\gamma_1+\frac{1}{\Theta}\frac{\partial\Theta}{\partial u}\right)\frac{\partial\rho}{\partial v}$$

If the asymptotic lines of the surface described by (ρ) to those of the given surface (ζ), we must have

$$a\frac{\Theta-\zeta\theta}{\Theta}=a$$

or

$$\zeta\theta=o$$

In this case the equations (VII) coincide with (I). This shows that the surface described by (ρ) is a projective transform of the given surface.

In fact, $\zeta=o$ is the case we have noticed in § 3. When $\theta=o$, the point (ρ) coincides with (x).

Hence we have the following theorem.

The asymptotic lines of a surface correspond to those of the surface which is in the relation (D) to it when and only when the latter is a projective transform of the former.

Let $\bar{a}$, $\bar{B}$ and $\bar{C}$ be the same functions as a, B and C respectively but formed with the coefficient of (VII).

Then we know that

$$\bar{a}=a\frac{\Theta-\zeta\theta}{\Theta}$$

$$\bar{B}=B+\frac{3}{8}\frac{\partial}{\partial v}log\,(\Theta-\zeta\theta)-\frac{1}{8}\frac{\partial}{\partial v}log\Theta$$

$$\bar{c}=c-\frac{1}{8}\frac{\partial}{\partial u}log\,(\Theta-\zeta\theta)+\frac{3}{8}\frac{\partial}{\partial u}log\Theta$$

from which follows

$$2\frac{\partial\bar{B}}{\partial u}-2\frac{\partial\bar{C}}{\partial v}-\frac{\partial^2}{\partial u\partial v}log\,\bar{a}=2\frac{\partial B}{\partial u}-2\frac{\partial C}{\partial v}-\frac{\partial^2}{\partial u\partial v}log\,a$$

We know easily

$$2\frac{\partial B}{\partial u}-2\frac{\partial c}{\partial v}-\frac{\partial^2}{\partial u\partial v}\log a=-2\left(H-K+\frac{\partial^2}{\partial u\partial v}\log a\right).$$

where H and K are Laplace-Darboux invariants, that is,

$$H=d_1+b_1c_1-\frac{\partial b_1}{\partial u}$$

$$K=d_1+b_1c_1-\frac{\partial c_1}{\partial v}$$

Wilczynski has called the line of the intersection of the osculating planes at (x) of the curves of the conjugate net on a surface (Σ) which meet at (x) the axis of the point (x) with respect to this conjugate net.

When the point (x) moves on the surface (Σ), the axes form a congruence which is called by him the axis congruence. The curves in which the developables of the axis congruence cut the surface (Σ) are called the axis curve.

Green[1] has proved that the condition that the axis curve with respect to the given conjugate net may also form a conjugate net is[1]

$$H-K+\frac{\partial^2}{\partial u\partial v}\log a=0$$

Hence we have the following theorem.

If the axis curves with respect to the given conjugate net form a conjugate net, the same is the case for all the surfaces which are in the relation (D) to it.

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[1] Amer. J. Math., 38, (1916).

Vol. V, No. 5, Published June 20th, 1922.

Price - - - Yen 1.30

These Memoirs are on sale at

MARUZEN COMPANY, LTD.

TOKYO, OSAKA, KYOTO, YOKOHAMA, NAGOYA, FUKUOKA & SENDAI.

大正十一年六月十七日印刷
大正十一年六月二十日發行

編纂兼發行者　京都帝國大學理學部

印刷者　大久保秀次郎
東京府下荏原郡世田谷村字下町五十番地

印刷所　株式會社 東京築地活版製造所
東京市京橋區築地二丁目十七番地

賣捌所　丸善株式會社
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